Universal relaxation of pinned density waves in Condensed Matter and Holography

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Rich dialogue between holography and effective theories: eg KSS bound, anomalies.

\[ \frac{\eta}{s} \gtrsim \frac{1}{4\pi} \]

First established in the context of asymptotically AdS black branes: Schwarzschild, RN, etc.

Sharp definition in the context of relativistic hydrodynamics: \( \eta \) quantifies the diffusion of transverse momentum

\[
G_{P \perp P \perp}^R (\omega, q) = \frac{\chi_{PP} D_\perp q^2}{i\omega - D_\perp q^2}, \quad D_\perp = \frac{\eta}{\chi_{PP}}
\]

Weaker bound \( \eta > 0 \) for positivity of entropy production.

Less symmetric cases? This talk: broken translations in 2d isotropic Wigner crystals.
Talk mostly based on

- ‘Diffusion and universal relaxation of holographic phonons’ [arXiv:1904.11445], JHEP’19, AAM

But see also

- ‘Bad Metals from Density Waves’ [arXiv: 1612.04381], Scipost’17, DHK
- ‘Theory of hydrodynamic transport in fluctuating electronic charge density wave states’ [arXiv:1702.05104], PRB’17, DHK
- ‘DC resistivity of quantum critical, charge density wave states from gauge-gravity duality’ [arXiv:1712.07994], PRL’18, AAM
- ‘Effective holographic theory of charge density waves’ [arXiv:1711.06610], PRD’18, AAM
- ‘Gapless and gapped holographic phonons’ [arXiv:1910.11330], AAM

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Related recent work:


- **Higher-form global symmetries**: Grozdanov and Poovuttikul [1801.03199], Armas and Jain [1908.01175].
• When translations are spontaneously broken, new gapless degrees of freedom are generated: the **Goldstone bosons** (phonons).

• New transport degrees of freedom: **constrained by positivity of entropy production**:

\[
\gamma_1^2 \leq \text{Max} \left( \sigma_o \Xi, \frac{\sigma_o \Omega}{\chi_{PP} \omega_o^2} \right)
\]

• Under certain circumstances, at low temperatures, these new bounds are saturated: **no entropy production**

\[
\gamma_1^2 = \text{Max} \left( \sigma_o \Xi, \frac{\sigma_o \Omega}{\chi_{PP} \omega_o^2} \right)
\]

• In a holographic toy model of broken translations
• In 2d electron gases in a large magnetic field (GaAs heterostructures)
• Holographic model: \( \Omega/\omega_o^2 = \chi_{PP} \Xi \)
The low energy dynamics of the ordered phase differ from those of the disordered phase by the necessity to include **new gapless degrees of freedom** (the Goldstones).

An important property of Goldstones is that they are **shift-symmetric**: they realize non-linearly the broken symmetry. More concretely, take broken translations along $x$

$$x \rightarrow x + c \quad \Rightarrow \quad \phi \rightarrow \phi + c$$

Shift symmetry: **only gradient terms** in the effective IR action:

$$f = \frac{1}{2} (K + G) \lambda_2 + \frac{1}{2} G \lambda_2 + \ldots$$

where $\lambda_\parallel = \nabla \cdot \vec{\phi}$, $\lambda_\perp = \nabla \times \vec{\phi}$. 
\[ f = \frac{1}{2} (K + G)\lambda^2 + \frac{1}{2} G\lambda^2 + \ldots \]

- \( K \) and \( G \) are the bulk and shear moduli:

\[
\chi_{\lambda_{\parallel} \lambda_{\parallel}} \equiv \frac{\delta \lambda_{\parallel}}{\delta s_{\parallel}} = \frac{1}{K + G}, \quad \chi_{\lambda_{\perp} \lambda_{\perp}} \equiv \frac{\delta \lambda_{\perp}}{\delta s_{\perp}} = \frac{1}{G}
\]

They are the static response to bulk compression and shear stress.
Since $\pi^i$ is the charge that generates the symmetry, then

$$[\varphi_i(x), \pi_j(y)] = i\delta_{ij}\delta(x - y) + \ldots$$

The effective Hamiltonian contains a term

$$H = \int d^d x \, v_i(x)\pi^i(x) + \ldots$$

which leads to the 'Josephson' relations

$$\dot{\lambda}_\parallel = \nabla \cdot \mathbf{v} + O(\nabla^2), \quad \dot{\lambda}_\perp = \nabla \times \mathbf{v} + O(\nabla^2)$$

At higher order in gradients (+relativistic symmetry), linear, diffusive couplings

$$\dot{\lambda}_\parallel = \nabla \cdot \mathbf{v} + \gamma_1 T \nabla^2 \left( \frac{\mu}{T} \right) + \xi_\parallel \nabla^2 \lambda_\parallel + O(\nabla^3),$$

$$\dot{\lambda}_\perp = \nabla \times \mathbf{v} + \xi_\perp \nabla^2 \lambda_\perp + O(\nabla^3)$$
Constitutive relation for the electric current

\[ j = \rho v - \sigma_o T \nabla \left( \frac{\mu}{T} \right) - \gamma_1 \nabla \lambda_\parallel \]

Isotropic crystal

\[ \frac{\xi_\parallel}{K + G} = \frac{\xi_\perp}{G} = \Xi \]

Bound ensuring positivity of entropy

\[ \gamma_1^2 \leq \sigma_o \frac{\xi_\parallel}{K + G} = \sigma_o \Xi \]
\[
S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{Z(\phi)}{4} F^2 - V(\phi) - Y(\phi) \left( \partial \psi_x^2 + \partial \psi_y^2 \right) \right]
\]

\[
Y(\phi) = \phi^2 + O(\phi^3), \quad Z(\phi) = 1 + O(\phi), \quad V(\phi) = -6 + \phi^2 + O(\phi^3)
\]

- **Homogeneous generalized Q-lattice Ansatz** \cite{Andrade & Withers'13, Donos & Gauntlett'13}: \( \psi_i = kx^i \). Breaks Translations × Global shifts to a diagonal U(1).

- **UV boundary conditions on \( \phi \)**

\[
\phi = \lambda r + \phi_v r^2 + \ldots
\]

- If \( \lambda = 0 \), then \( \psi_i = kx^i \) is a vev: **spontaneous breaking**.

- If \( \lambda \neq 0 \), then \( \psi_i = kx^i \) is a source: **explicit breaking**.

- But if \( \lambda/\mu \ll \phi_v/\mu^2 \), **pseudo-spontaneous breaking**.
We compute $\gamma_1$, $\sigma_o$ and $\Xi$ holographically (using radially conserved bulk currents). The positivity of entropy production bound is obeyed

$$\sigma_o \Xi - \gamma_1^2 \geq 0$$

Even saturates at low $T$. Why?
From

\[ G^R_{J\varphi\parallel}(\omega, q = 0) = \gamma_1 + \frac{i\rho}{\chi_{PP}\omega}, \]
\[ G^R_{\varphi\parallel\varphi\parallel}(\omega, q = 0) = \frac{1}{\chi_{PP}\omega^2} - \Xi \frac{i}{\omega}, \]

we can write Kubo formulæ

\[ \gamma_1 = \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G^R_{J\varphi}(\omega, q = 0), \]
\[ \Xi = \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G^R_{\varphi\varphi}(\omega, q = 0). \]

All we need is a mechanism producing a nonzero \( \partial_t \varphi \).
The saturation at low temperatures can be explained by **universal relaxation into the heat current**

\[ \Delta H = \int dx \frac{\pi \cdot j_q}{\chi_{Pj_q}} \quad \Rightarrow \quad \dot{\varphi} = \frac{j_q}{\chi_{Pj_q}} \]

\[ \gamma_1 = \frac{1}{\chi_{Pj_q}} \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G_{jj_q}^R(\omega, q = 0) = -\frac{\mu}{\chi_{Pj_q}} \sigma_o, \]

\[ \Xi = \left( \frac{1}{\chi_{Pj_q}} \right)^2 \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G_{j_qj_q}^R(\omega, q = 0) = \left( \frac{\mu}{\chi_{Pj_q}} \right)^2 \sigma_o. \]

These values verify \( \gamma_1^2 = \sigma_o \Xi \) and match our numerics.
Now, break translations **explicitly, weakly**.

The Goldstones become **massive** and **damped**

\[
f = \frac{1}{2}(K + G)(\nabla \cdot \varphi)^2 + \frac{1}{2}G(\nabla \times \varphi)^2 + \frac{1}{2}m^2\varphi^2 + \ldots
\]

\[
\dot{\varphi}^i = -\Omega \varphi^i + O(q)
\]

Also relaxes momentum

\[
\dot{\pi}^i = -\Gamma \pi^i - Gm^2\varphi^i + \ldots
\]

All poles are gapped

\[
(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2 = 0
\]

with \(\omega_o \equiv m\sqrt{(G/\chi_{pp})}\) the pinning frequency.
There is another positivity of entropy production bound

\[ \gamma_1^2 \leq \frac{\Omega \sigma_o}{\chi_{PP} \omega_o^2} \]

- **Almost saturates**: relaxation of the weakly-gapped phonons into the heat current.
- **Violation at low temperature**: breakdown of WC hydro picture (the phonons become very strongly damped).
Now turn on a magnetic field: the longitudinal and transverse sound modes hybridize into (gapless) magnetophonons and gapped magnetoplasmons.

Upon turning on disorder, the magnetophonons are pinned at $\omega_0^2/\omega_c \sim O(1/B)$: within hydrodynamics at large magnetic fields.

Write down a similar hydrodynamic theory as before: conservation of charge, Josephson for magnetophonon, constitutive relations, solve and get conductivity.

Positivity of entropy production bound.
We want to break magnetic translations. When broken generators do not commute, reduction on the number of expected Goldstones [Watanabe & Muruyama’12].

Under magnetic translations, Goldstones $\varphi_i \rightarrow \varphi_i + \delta x_i$. Leads to

$$\mathcal{L} = \epsilon^{ij} \varphi_i \dot{\varphi}_j$$

Upon quantizing

$$[\varphi_i(x), \varphi_j(y)] = -i \epsilon_{ij} \delta(x - y)$$

The Goldstones are not independent fields!

From Noether, conserved densities are $\pi^i \sim \epsilon^{ij} \varphi_j$, which leads to the magnetic translation algebra:

$$\Rightarrow [\mathcal{P}_i, \mathcal{P}_j] = -iRB\epsilon_{ij}$$
The free energy including pinning and leading spatial gradients

\[ \mathcal{L} = \epsilon^{ij} \varphi_i \dot{\varphi}_j - \varphi_j \left[ \delta^{ij} \omega_{pk} + (Kk^i k^j + Gk^2 \delta^{ij}) + \ldots \right] \varphi_j \]

Leads to the modes [Fukuyama & Lee’78]

\[ \omega(k) = \pm \sqrt{\left( \omega_{pk} + Gk^2 \right) \left( \omega_{pk} + (K + G)k^2 \right)} \]

\[
\begin{cases} 
\omega_{pk} = 0 & \Rightarrow \omega(k) = \pm k^2 \\
 k = 0 & \Rightarrow \omega = \pm \omega_{pk}
\end{cases}
\]
Enters relaxation

\[
\begin{pmatrix}
  j^i \\
  \varphi^i
\end{pmatrix} =
\begin{pmatrix}
  \sigma_{ij}^o & \gamma^{ij} \\
  \gamma^{ij} & \Omega^{ij}/\omega_{pk}
\end{pmatrix}
\begin{pmatrix}
  E_j \\
  s_j - \omega_{pk}\varphi_j
\end{pmatrix}
\]

\[
\sigma_{ij}^o = \sigma_o \delta^{ij} + \sigma_o^H \epsilon^{ij}, \gamma^{ij} = \gamma \delta^{ij} + \sqrt{\nu} \epsilon^{ij}, \Omega^{ij} = \Omega \delta^{ij} + \omega_{pk} \epsilon^{ij}
\]

Conductivity

\[
\sigma_{xx}(\omega) = \sigma_o + \nu \omega_{pk} \frac{(1 - a^2)(-i\omega + \Omega) - 2a\omega_{pk}}{(-i\omega + \Omega)^2 + \omega_{pk}^2} \quad \nu = \frac{\rho}{B}.
\]

new: \( a \equiv \gamma/\sqrt{\nu} \) asymmetry parameter.

Positivity of entropy production:

\[
\gamma^2 \leq \frac{\sigma_o \Omega}{\omega_{pk}}
\]
Fit to data on GaAs heterojunctions (2DEG) \cite{YP Chen et al, Nature Physics’06}, \cite{YP Chen et al, International Journal of Modern Physics B’07}, \cite{YP Chen, PhD thesis’05}

Sample A
Thermal melting

Sample B
Quantum melting

Sample C
Thermal melting (more disordered)
Ω increases as melting is approached: shorter-lived magnetophonon.
The fits require a **nonzero asymmetry parameter** $a \neq 0$:
- Compute relaxation parameters? Use Kubo formulas

\[ \Omega = \omega_{pk} \lim_{\omega \to 0} \lim_{\Omega, \gamma \to 0} \frac{1}{\omega} \text{Im} G_{\phi_x \phi_x}^R (\omega), \]

\[ \gamma = \lim_{\omega \to 0} \lim_{\Omega, \gamma \to 0} \frac{1}{\omega} \text{Im} G_{j_x \phi_x}^R (\omega), \]

- Now need to compute $\dot{\phi}$.

- **Mobile dislocations**

\[ \Omega_{\text{vor}} = \frac{2x}{\sigma_n} \nu \omega_{pk}, \quad \gamma_{\text{vor}} = x \sqrt{\nu} \frac{\sigma_n^H}{\sigma_n} \quad \Rightarrow \quad \frac{\Omega}{a \omega_{pk}} = 2. \]

- **Relaxation into current**

\[ H_{\text{dis}} = \frac{1}{\sqrt{\nu}} \int d^2x \epsilon_{ij} \phi_i (x) j_j (x). \]

\[ \Omega_{\text{dis}} = \frac{\omega_{pk} \sigma_0}{\nu}, \quad \gamma_{\text{dis}} = \frac{\sigma_0}{\sqrt{\nu}} \quad \Rightarrow \quad \frac{\Omega}{a \omega_{pk}} = 1. \]
Different microscopic relaxation mechanisms appear to be at play in the different samples:

Samples A and B
mobile dislocations

Sample C (more disordered)
universal dissipation into hydrodynamic currents
We observed the saturation of two bounds on positivity of entropy production

\[ \gamma^2 \leq \sigma_o \Xi, \quad \gamma^2 \leq \frac{\sigma_o \Omega}{\chi_{PP}\omega_o^2} \]

Explained by universal relaxation of Goldstones into the heat current. Is this generic? See also [Davison, Schalm, Zannen’13], [Lucas’15].
\[ \gamma^2 \leq \sigma_0 \Xi, \quad \gamma^2 \leq \frac{\sigma_0 \Omega}{\chi_{PP} \omega_0^2} \]

- This suggests a relation between the Goldstone relaxation rate and mass should hold at low enough \( T \)

\[ \Omega = \chi_{PP} \omega_0^2 \Xi \]

- In the holographic model, we can actually show it is a consequence of weak explicit breaking of translations, rather than low temperature. It holds even at higher temperatures, where the entropy bounds are far from saturation.

- Universal contribution to the damping rate? Artefact of the holographic toy model? Other holographic models?
An argument for universality

\[ \partial_t \varphi = v + \Xi \nabla^2 s_\varphi \]

Spontaneous: \( s_\varphi = Gk^2 \varphi \)

Pseudo-spontaneous: \( s_\varphi = G(k^2 + m^2) \varphi \)

\[ \partial_t \varphi = v + G\Xi \nabla^2 \varphi + Gm^2 \Xi \varphi \]

\[ \Rightarrow \Omega = Gm^2 \Xi \]

In which circumstances is this the dominant contribution to the damping rate \( \Omega \)?

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