Infrared instability of chiral diffusion

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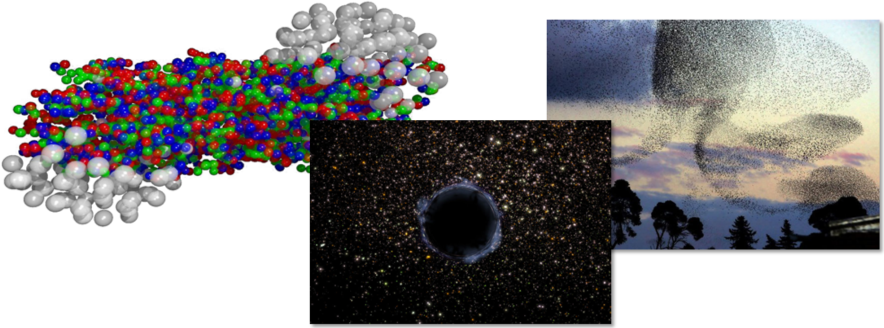
with
Hong Liu, Srivatsan Rajagopal [1710.03768]
review: Hong Liu [1805.09331]
Luca Delacretaz, *in progress*
Andrey Gromov and Shinsei Ryu [1908.03217]

Hydrodynamics provides the low-energy description of a huge variety of phenomena in nature. These range from heavy ion collisions to black holes dynamics, from driven systems to non-equilibrium steady states, etc.



In its most traditional form, hydrodynamics is the effective theory for conservation laws.

In this talk: Hydrodynamics for a conserved current with chiral anomaly, (mostly) in 1+1 dimensions

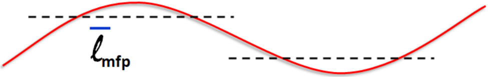
$$\partial_\mu J^\mu = -2\hbar c E$$

where E: electric field, c: anomaly coefficient.

- Chiral anomaly is a quantum effect. Technically, it is broken by the measure of the (quantum) path integral

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{\frac{i}{\hbar} S_0[\psi,\bar{\psi}]}$$

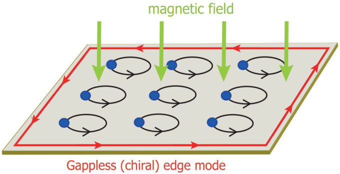
Hydrodynamic approach:



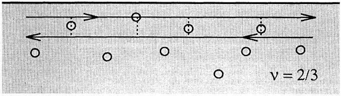
- Neglect energy-momentum conservation
- Local equilibrium: $\rho = e^{-\frac{1}{T}(H-\mu(t,x)Q)}$

Motivations I

- Edge of quantum Hall systems [Kane,Fisher '95; Ma,Feldman '19]

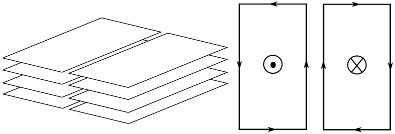


[Hasebe, Totsuka '13]



[Kane,Fisher '95]

- Surface chiral metals [Sur,Lee '13]



[Sur, Lee '13]

- Chiral magnetic effect [Vilenkin '80] [Son,Spivak '13] [Yamamoto '15]

$$\vec{J} \propto \mu \vec{B}$$

Chiral diffusion

$$\partial_\mu J^\mu = -2cE, \qquad (\text{set } \hbar = 1)$$

where E: electric field, c: anomaly coefficient.

$$J^t = n(\mu) = \chi\mu + \frac{1}{2}\chi'\mu^2 + \cdots, \quad J^x = -4c\mu - \sigma\partial_x\mu$$

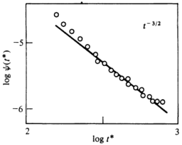
- $-4c\mu$ required by second law [Son,Surowka '09].
- Chiral diffusion:

$$\chi\partial_t\mu - 4c\partial_x\mu - \sigma\partial_x^2\mu + \frac{1}{2}\chi'\partial_t\mu^2 = 0$$

Motivations II

Hydrodynamic long time tails:

- Change qualitatively correlation functions at late time.



[Boon, "Molecular hydrodynamics," '91]

- Changes analytic structure of high-temperature correlation functions [Chen-Lin,Delacretaz,Hartnoll '18]
- Momentum conservation causes more violent effects [Forster,Nelson,Stephen '74], [Kovtun,Yaffe '03]. E.g. $d = 2$:

$$\langle J(\omega)J(-\omega)\rangle_{\text{sym}} \sim \sigma - \frac{T\chi}{w(D+\gamma_\eta)} \log\left(\frac{\omega}{\Lambda}\right)$$

Breakdown of hydrodynamics! [Schepper,Beyeren '74]

I will show:

- Chiral diffusion breaks down in the IR
- It persists even without momentum conservation!
- It furnishes a novel way to flow to a non-trivial IR fixed point.

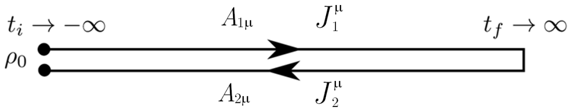
EFT of chiral diffusion

- Consider a quantum system in a thermal state $\rho_0 = e^{-\beta H}/\mathrm{Tr}(e^{-\beta H})$ with

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu}F_{\mu\nu}$$

- Background sources: $A_{1\mu}, A_{2\mu}$

$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(A_1)\rho_0 U^\dagger(A_2)\right] = \int_{\rho_0} D\psi_1 D\psi_2 e^{iS[\psi_1,A_1]-iS[\psi_2,A_2]}$$



- Anomalous conservation of J_1^μ and J_2^μ implies the Ward identity

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + 2c\int \lambda_1 F_1 - 2c\int \lambda_2 F_2$$

EFT of chiral diffusion

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + c\int \lambda_1 F_{1\mu\nu} - c\int \lambda_2 F_{2\mu\nu}$$

- W is non-local due to long-living modes associated to $\partial_\mu J_1^\mu = 0$ and $\partial_\mu J_2^\mu = 0$.
- "Unintegrate" long-living modes [Haehl,Loganayagam,Rangamani '15; Crossley, PG, Liu '15; Jensen, Pinzani-Fokeeva, Yarom '17;...]

$$e^{iW[A_1,A_2]} = \int D\varphi_1 D\varphi_2\, e^{iS_{\text{hydro}}[A_1,\varphi_1;A_2,\varphi_2]}$$

$$\varphi_1, \varphi_2 : \text{ long living modes}$$

- S_{hydro} local, satisfies several symmetries. Precisely recovers diffusion in the saddle-point limit.

IR instability

Minimal action for chiral diffusion:

$$S = \int d^2x \left(- \left(\chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t(\mu^2) \right) \varphi_a + iT\sigma(\partial_x \varphi_a)^2 \right)$$

where $\mu = \partial_t \varphi_r$ is the chemical potential, and

$$\begin{aligned} \varphi_r &= \frac{1}{2}(\varphi_1 + \varphi_2) \quad \text{classical variable} \\ \varphi_a &= \varphi_1 - \varphi_2, \quad \text{noise variable} \end{aligned}$$

At tree-level, this action recovers:

$$\partial_\mu J^\mu = \chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t(\mu^2) = 0$$

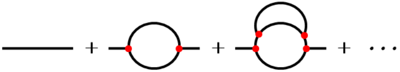
IR instability

It is convenient to change coordinates to a frame co-moving with the chiral front: $x \to x + \frac{4c}{\chi} t$. Upon rescaling various quantities:

$$S = \int d^2x \left(- \left(\partial_t \mu - \partial_x^2 \mu + \lambda \partial_x (\mu^2) \right) \varphi_a + i (\partial_x \varphi_a)^2 \right) , \quad \lambda = \frac{2c\chi' T}{\chi \sigma}$$

Scaling $\partial_t \sim \partial_x^2$, the interaction λ is relevant! This has dramatic consequences:

$$\langle J^i(\omega) J^i(-\omega) \rangle_{\text{ret}} \sim \sigma i\omega + \lambda^2 (i\omega)^{-\frac{1}{2}} + \lambda^4 (i\omega)^{-1} + \cdots$$



Correlation function grows with time!

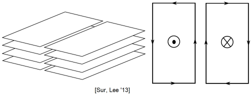
Fate in the IR

What is the fate of chiral diffusion in the IR?
To get a sense, consider higher-dimensional generalization:

$$J^x = -4c\mu - \sigma\partial_x\mu, \qquad J^\perp = -\sigma_\perp\nabla_\perp\mu$$

Rescaled coupling λ is marginal in $2+1$ and irrelevant in $3+1$.

- $(2+1)-d$: surface chiral metals



- $(3+1)-d$: chiral magnetic effect with large background magnetic field.

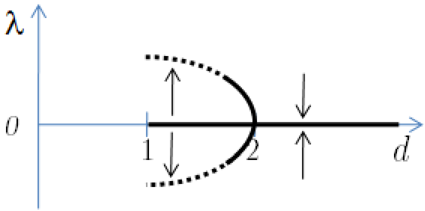
$$J^i \propto \mu B^i$$

Fate in the IR

Integrate out momentum shell $e^{-l}\Lambda < |k| < \Lambda$:

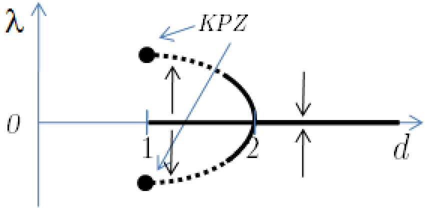
$$\frac{\partial \lambda}{\partial l} = \frac{1}{2}\varepsilon\lambda - \frac{\lambda^3}{2\pi}, \qquad \varepsilon = 2 - d$$

The theory is marginally irrelevant in $d = 2$ and has a non-trivial fixed point at $\varepsilon = 2 - d > 0$!



Fate in the IR

In $1+1$ dimensions, the theory flows to the KPZ (Kardar-Parisi-Zhang) universality class.



- Diffusive fluctuations around the chiral front at $x + \frac{4c}{\chi} t$ are in the KPZ universality class.
- Chiral diffusion flows to $\omega = \frac{4c}{\chi} k + k^z$, $z = \frac{3}{2}$, leading to the exact scaling

$$\sigma(\omega) = \langle J^i(\omega) J^i(-\omega) \rangle_{\mathrm{sym}} \sim \frac{1}{\omega^{1/3}}$$

Remarks

- Chiral diffusion has a non-trivial IR fixed point
- Precisely due to chiral anomaly
- Persists without momentum conservation

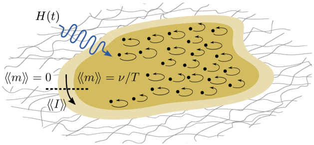
Future directions

1. Heat transport
2. More general non-equilibrium situations?

Advertisement:

Application of Schwinger-Keldysh effective field theories to far from equilibrium systems:

Topological response of periodically driven (Floquet) systems



[Nathan et al., '16]

- Inherently far from equilibrium.
- Topological field theory, e.g. for "chiral Floquet drive":

$$e^{iW[A_1,A_2]} = e^{i\frac{\Theta(\alpha)}{2\pi}\kappa\int d^2x[dA_1(x)-dA_2(x)]}$$

[PG,Gromov,Ryu 1908.03217]