HYDRODYNAMIC FRAMEWORK FOR VISCOELASTICITY

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[1908.01175, 1811.04913] J Armas, AJ

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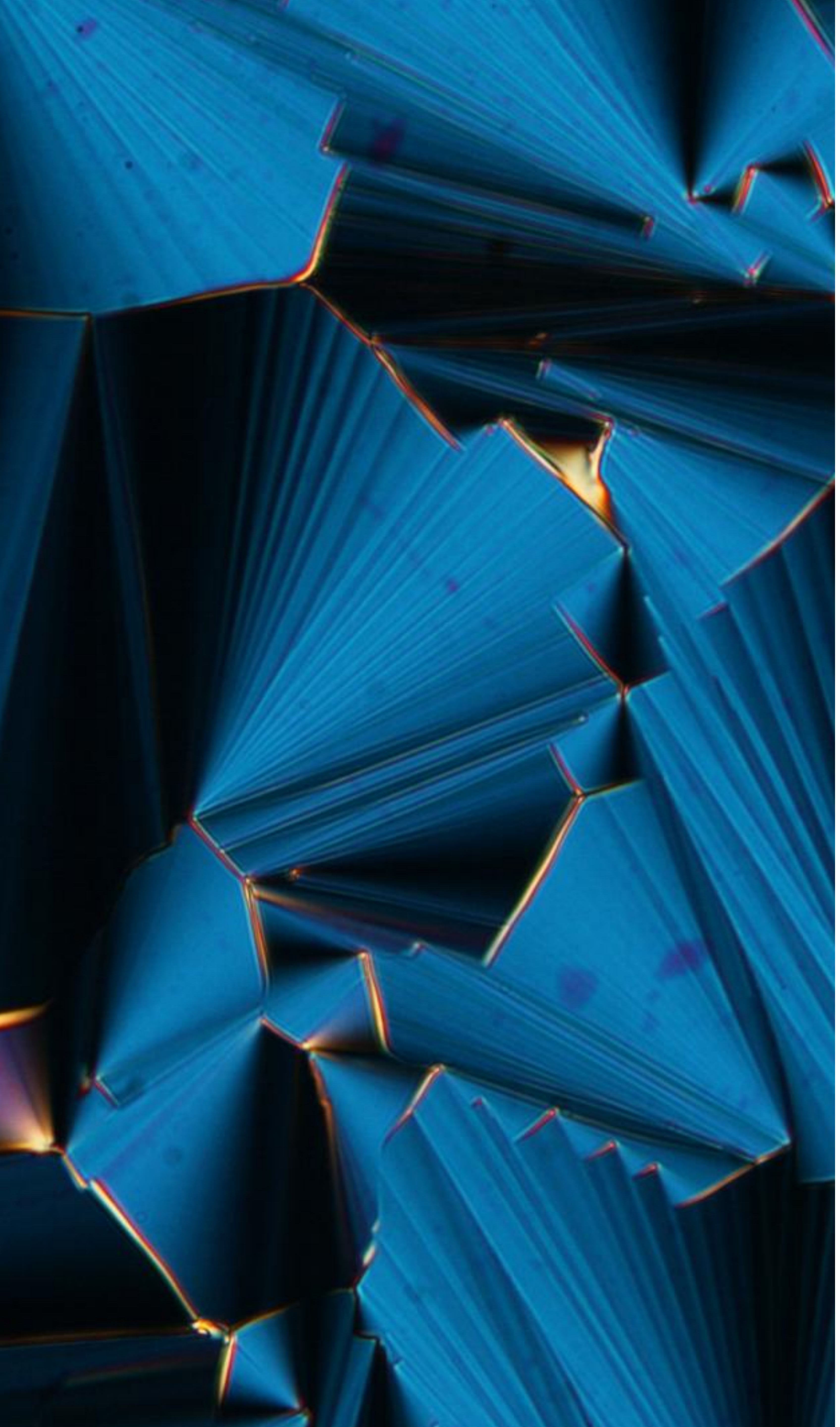


MOTIVATION

- Generalised global symmetries are a generic feature of theories with topologically conserved charges [1].
- Generalised symmetries are associated to higher-dimensional conserved objects like strings or surfaces.
- Magnetohydrodynamics can be reformulated as a theory of conserved strings (magnetic field lines). The dual theory is a special case of one-form superfluids [2].
- Superfluid dynamics is equivalent to a fluid with an anomalous (d-1)-form symmetry [3].
- Viscoelastic hydrodynamics can be understood as a theory with d copies of (d-1)-form symmetries [4].

[1] Gaiotto, Kapustin, Seiberg, Willett [1412.5148]
[2] Grozdanov, Hofman, Iqbal [1610.07392]; Armas, AJ [1803.00991,1808.01939,1811.04913]
[3] Delacretaz, Hofman, Mathys [1908.06977]
[3] Grozdanov, Poovuttikul [1801.03199]; Armas, AJ [1908.01175]

CLASSICAL THEORY OF ELASTICITY

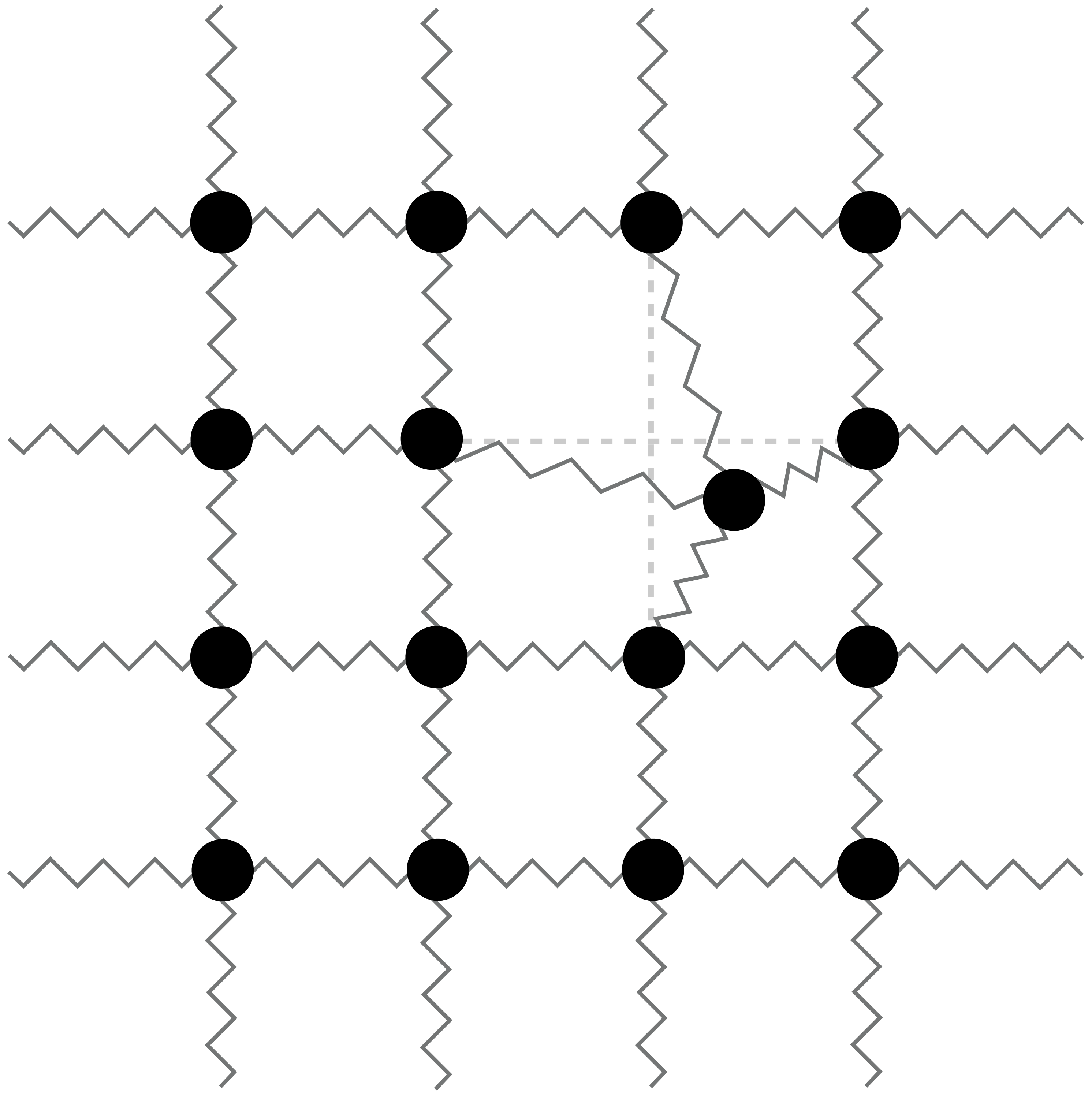
- Deformation: $x^i \to x^i + \delta x^i(x)$
- Free energy:

$$F = \int d^3x \left(\frac{1}{2} C^{ijkl}\, \partial_i \delta x_j\, \partial_k \delta x_l \right)$$

$$C^{ijkl} = B\, \delta^{ij}\delta^{kl} + 2G \left(\delta^{i(k}\delta^{l)j} - \frac{1}{3}\delta^{ij}\delta^{kl} \right)$$

- Stress tensor:

$$T^{ij} = -\overbrace{B}^{\text{Bulk Modulus}}\, \delta^{ij}\, \partial_k \delta x^k - \underbrace{2G}_{\text{Shear Modulus}} \left(\partial^{(i}\delta x^{j)} - \frac{1}{3}\delta^{ij}\, \partial_k \delta x^k \right)$$



[1] Chaikin, Lubensky - "Principles of Condensed Matter Physics".

SIGMA MODEL FOR ELASTICITY

- Spatial arrangement of lattice sites: $\phi^I(x)$
- Induced metric:

$$h^{IJ} = \eta^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J$$

- Free energy:

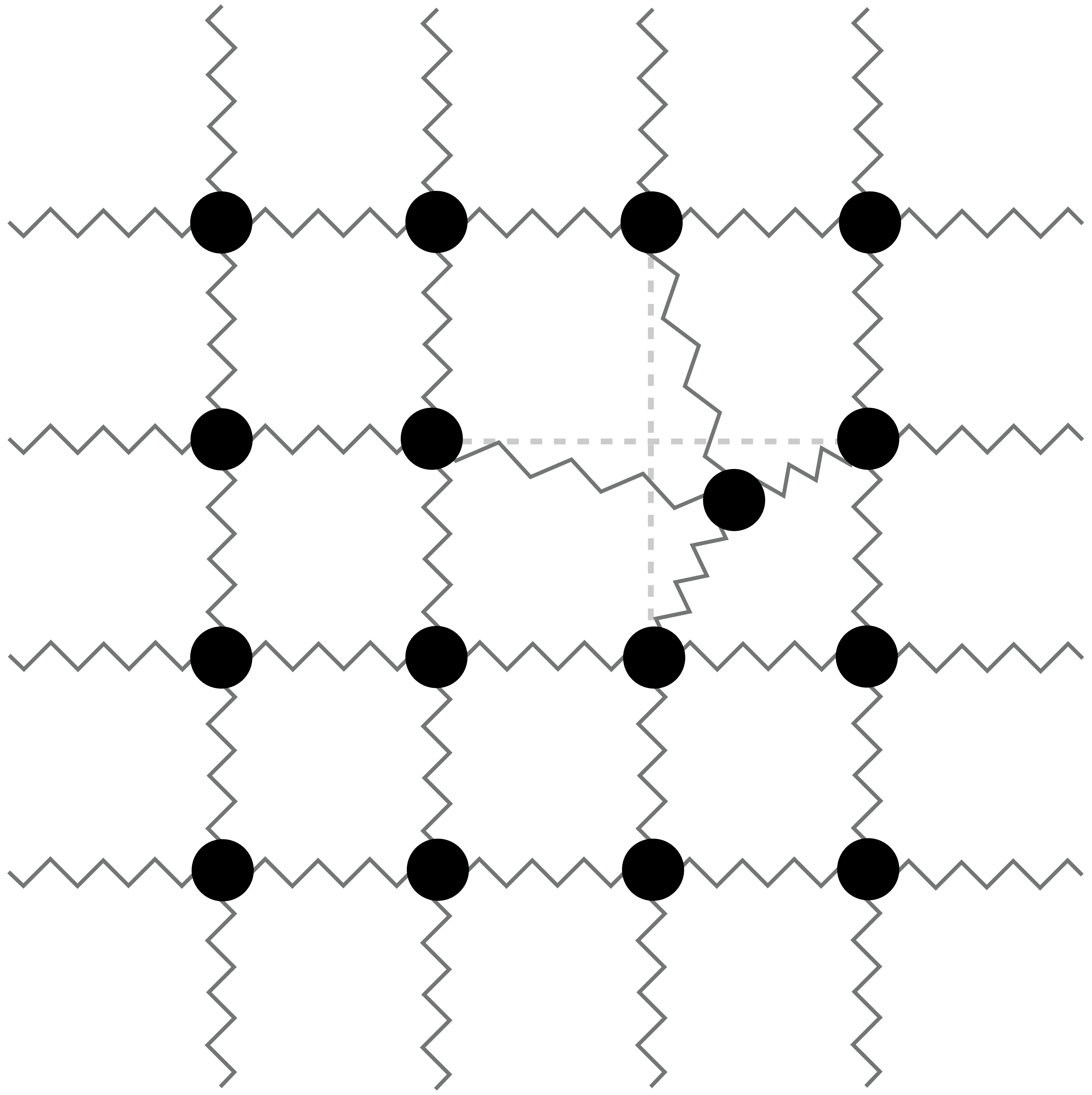
$$F = -\int d^3x\, P(h^{IJ})$$

- Stress tensor:

$$T^{ij} = P\,\delta^{ij} - 2\frac{\partial P}{\partial h^{IJ}}\partial^i\phi^I\partial^j\phi^J$$

- Configuration equation:

$$\frac{\delta F}{\delta\phi^I} = \partial_\mu\left(2\frac{\partial P}{\partial h^{IJ}}\partial^\mu\phi^J\right) = 0$$



$\phi^I = \delta^I_i x^i$ is a trivial solution.

LINEAR ELASTICITY AND LATTICE PRESSURE

- Linear deformation: $\phi^I(x) = \delta^I_i \left(x^i - \delta x^i(x)\right)$
- Strain tensor:

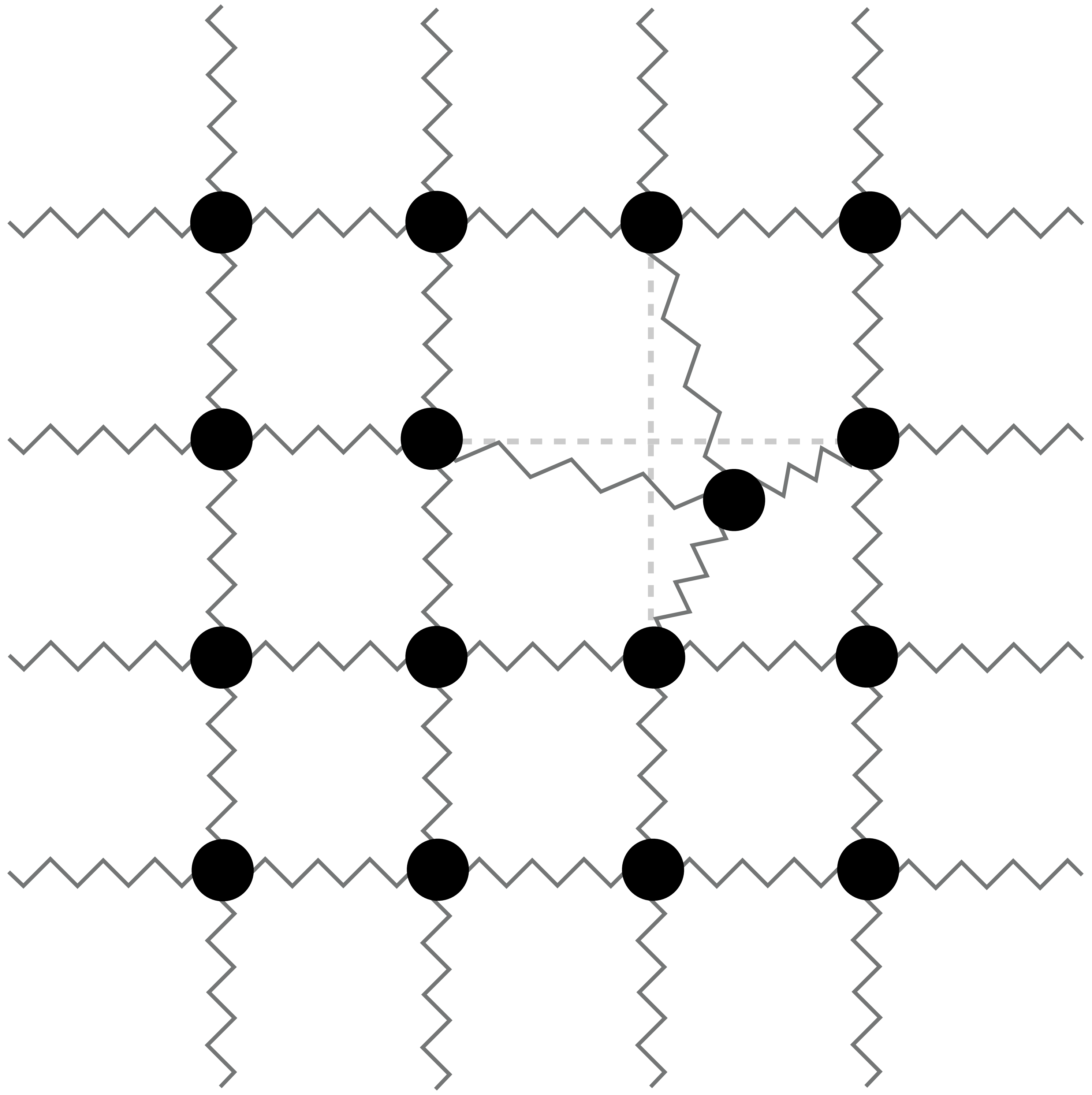
$$\varepsilon_{IJ} = \frac{1}{2}\left(h_{IJ} - \delta_{IJ}\right) \sim \delta^i_I \delta^j_J \, \partial_{(i}\delta x_{j)} + \dots$$

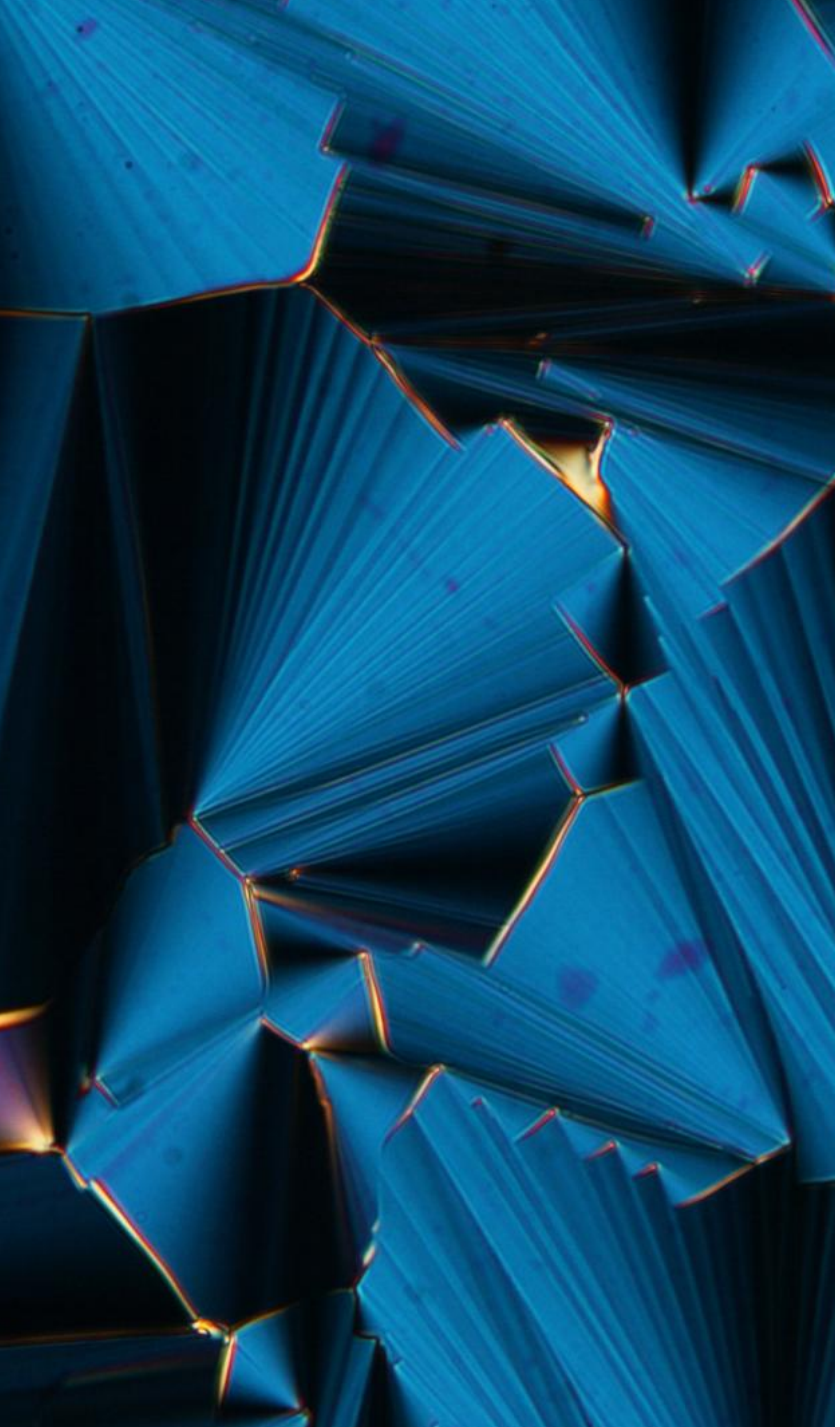
- Pressure:

$$P(h^{IJ}) = P_f + P_\ell \, h^{IJ} \varepsilon_{IJ} - \frac{1}{2} C^{IJKL} \varepsilon_{IJ} \varepsilon_{KL} + \dots$$

- Linear deformation:

$$T^{ij} = \overbrace{P_f \, \delta^{ij}}^{\text{Thermodynamic Pressure}} + \underbrace{P_\ell \, \delta^{ij}}_{\text{Lattice Pressure}} - B \, \delta^{ij} \, \partial_k \delta x^k - 2G \left(\partial^{(i}\delta x^{j)} - \frac{1}{3} \delta^{ij} \, \partial_k \delta x^k \right)$$





PLAN

- Viscoelasticity as hydrodynamics with spontaneously broken translations
- Generalised global symmetries in viscoelasticity
- Holographic models for viscoelasticity
- Outlook

HYDRODYNAMICS AND VISCOELASTICITY

- Dynamical fields:

$$u^\mu \quad (u^\mu u_\mu = -1), \qquad T, \qquad \phi^I$$

Dynamical equations:

$$\partial_\mu T^{\mu\nu} = 0, \qquad K_I = 0$$

e.g. Josephson equation for superfluids, Young-Laplace equation for fluid surfaces, or Maxwell's equations for magnetohydrodynamics.

- Constitutive relations:

$$T^{\mu\nu}[u, T, \phi], \qquad K_I[u, T, \phi] = 0$$

Adiabaticity equation/Second law of thermodynamics [1]:

$$\partial_\mu N^\mu - T^{\mu\nu}\partial_\mu \frac{u_\nu}{T} - K_I \frac{u^\mu}{T}\partial_\mu \phi^I \geq 0 \quad \Longleftrightarrow \quad \begin{gathered} \partial_\mu S^\mu \geq 0 \quad [onshell] \\ S^\mu = N^\mu - \frac{1}{T}T^{\mu\nu}u_\nu \end{gathered}$$

[1] R. Loganayagam [1106.0277], AJ [1610.05797]

ONE DERIVATIVE VISCOELASTIC FLUIDS

- Energy-momentum tensor

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, P^{\mu\nu} - r_{IJ}\, \partial^\mu \phi^I \partial^\nu \phi^J - \eta_{IJKL}\, P^{I\mu} P^{J\nu} P^{K\rho} P^{L\sigma} \partial_\rho u_\sigma$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

$$P^{I\mu} = P^{\mu\nu} \partial_\nu \phi^I$$

$$h^{IJ} = \eta^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J$$

$$dP = s dT + \frac{1}{2} r_{IJ} dh^{IJ}, \qquad \epsilon = sT - P \qquad \Longrightarrow \qquad d\epsilon = T ds - \frac{1}{2} r_{IJ} dh^{IJ}$$

- Configuration equation

$$K_I = -\, \sigma_{IJ}\, u^\mu \partial_\mu \phi^J - \partial_\mu \left(r_{IJ}\, \partial^\mu \phi^J \right) = 0 \qquad \Longrightarrow \quad u^0 \partial_t \phi^I = -\, \delta^I_i u^i - u^i \partial_i \delta \phi^I + \ldots$$

- Second law of thermodynamics

$$T \partial_\mu S^\mu = \eta_{IJKL} \left(P^{I\mu} P^{J\nu} \partial_\mu u_\nu \right) \left(P^{K\rho} P^{L\sigma} \partial_\rho u_\sigma \right) + \sigma_{IJ} \left(u^\mu \partial_\mu \phi^I \right) \left(u^\nu \partial_\nu \phi^J \right) \geq 0$$

LINEAR VISCOELASTIC FLUIDS

- Expanding coefficients:

$$P = P_f + P_\ell\, h^{IJ}\varepsilon_{IJ} - \frac{1}{2}B\,(h^{IJ}\varepsilon_{IJ})^2 - G\,h^{I\langle K}h^{L\rangle J}\varepsilon_{IJ}\varepsilon_{KL} + \ldots$$

$$\eta_{IJKL} = 2\eta\, h_{I\langle K}h_{L\rangle J} + \zeta\, h_{IJ}h_{KL} + \ldots, \qquad \sigma_{IJ} = \sigma\, h_{IJ} + \ldots$$

$$\varepsilon_{IJ} = \frac{1}{2}\left(h_{IJ} - \delta_{IJ}\right)$$

$$\varepsilon_{\mu\nu} = P^I_\mu P^J_\nu \varepsilon_{IJ}$$

$$P^{I\mu} = P^{\mu\nu}\partial_\nu \phi^I$$

$$\sigma^{\mu\nu} = 2P^{\mu\rho}P^{\nu\sigma}\partial_{\langle\rho}u_{\sigma\rangle}$$

- Energy-momentum tensor [1]:

$$T^{\mu\nu} = \epsilon_f\, u^\mu u^\nu + P_f\, P^{\mu\nu} - \eta\,\sigma^{\mu\nu} - \zeta\, P^{\mu\nu}\partial_\lambda u^\lambda$$
$$+ P_\ell\, P^{\mu\nu} + \epsilon_\ell\, \varepsilon^\lambda{}_\lambda\, u^\mu u^\nu - 2G\,\varepsilon^{\langle\mu\nu\rangle} - B\,\varepsilon^\lambda{}_\lambda P^{\mu\nu} + \ldots$$

$$dP_f = s_f dT, \qquad \epsilon_f = s_f T - P_f \qquad \Longrightarrow \qquad d\epsilon_f = T ds_f \qquad\qquad \epsilon_\ell = TP'_\ell - P_\ell$$

- 5 new transport coefficients in energy-momentum tensor:

$$T^{\mu\nu} = \ldots - \eta^u_1\,\varepsilon^\lambda{}_\lambda\sigma^{\mu\nu} - \eta^u_2\,\varepsilon^{\langle\mu}{}_\lambda\sigma^{\nu\rangle\lambda} - \zeta^u_1\, P^{\mu\nu}\varepsilon^\lambda{}_\lambda\partial_\lambda u^\lambda - (\zeta^u_2 + \bar\zeta^u)\, P^{\mu\nu}\varepsilon_{\mu\nu}\sigma^{\mu\nu} - (\zeta^u_2 - \bar\zeta^u)\,\varepsilon^{\langle\mu\nu\rangle}\partial_\lambda u^\lambda + \ldots$$

Another new transport coefficient in configuration equation.

[1] Martin, Parodi, Pershan; Jahnig, Schmidt 1972; Fukuma, Sakatani [1104.1416,1204.6288]

LINEAR MODES

- Longitudinal sound:

$$\omega = \pm k \underbrace{\sqrt{\frac{(w+\epsilon_\ell)^2}{T^2 s' w} + \frac{B + \frac{4}{3}G - P_\ell}{w}}}_{v_\parallel} - \frac{ik^2}{2}\underbrace{\left(\frac{T^2 s^2}{\sigma w v_\parallel^2}\left(v_\parallel^2 - \frac{w+\epsilon_\ell}{T^2 s'}\right)^2 + \frac{\zeta + \frac{4}{3}\eta}{w}\right)}_{\Gamma_\parallel} + \dots$$

- Transverse sound:

$$\omega = \pm k \underbrace{\sqrt{\frac{G}{w}}}_{v_\perp} - \frac{ik^2}{2}\underbrace{\left(\frac{T^2 s^2 G}{\sigma w^2} + \frac{\eta}{w}\right)}_{\Gamma_\perp} + \dots$$

$$w = Ts + P_\ell$$

$$\epsilon_\ell = TP'_\ell - P_\ell$$

- Diffusive mode:

$$\omega = -ik^2 \underbrace{\left(\frac{s^2}{\sigma s' v_\parallel^2}\frac{B + \frac{4}{3}G - P_\ell}{w}\right)}_{D_\parallel} + \dots$$

GENERALISED GLOBAL SYMMETRIES IN VISCOELASTICITY

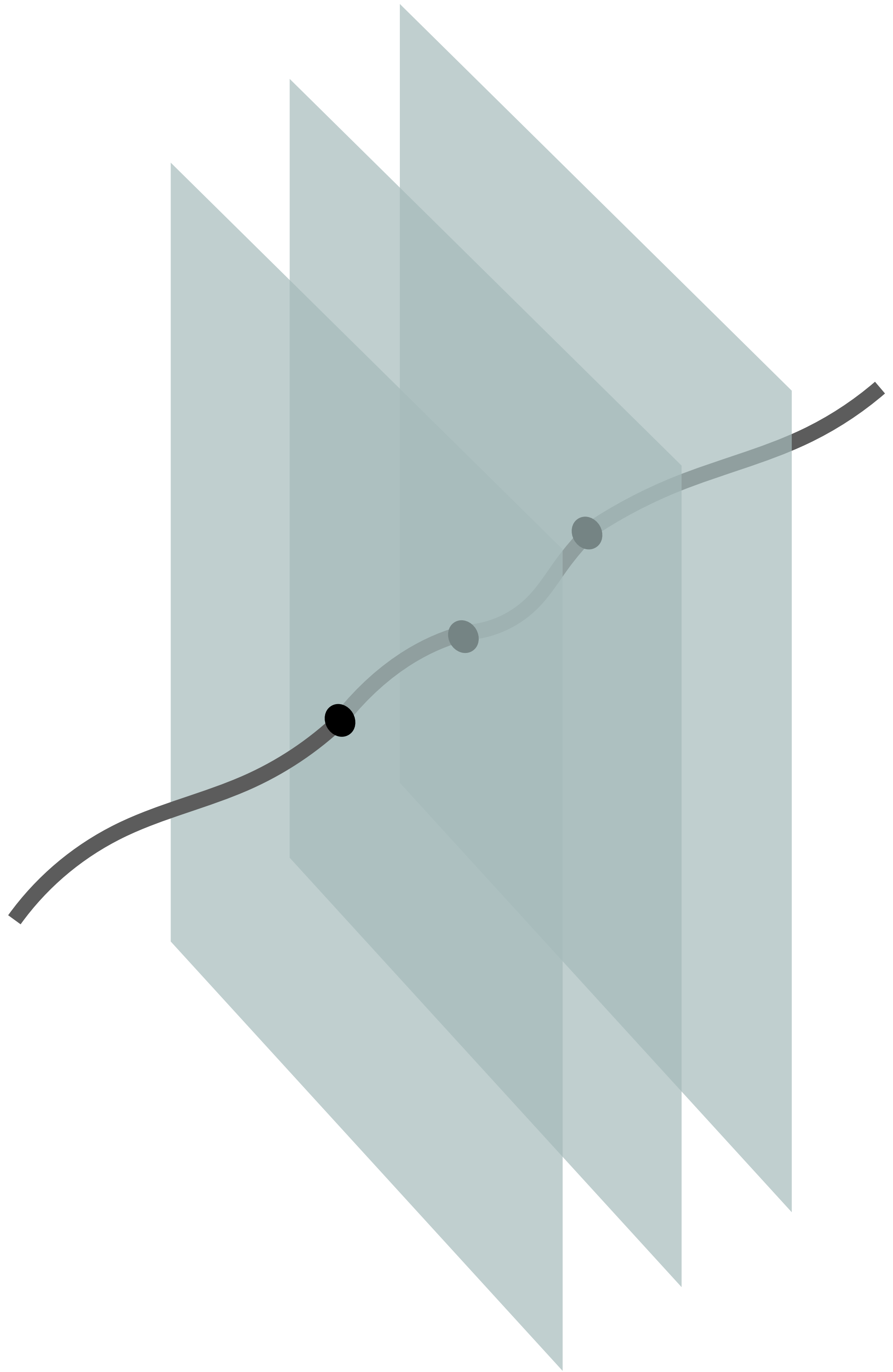
- Bianchi identity [1]

$$\partial_\mu J^{I\mu\nu\rho} = 0, \qquad J^{I\mu\nu\rho} = \epsilon^{\lambda\mu\nu\rho}\partial_\lambda\phi^I = \epsilon^{\lambda\mu\nu\rho}\left(P^I_\lambda - u_\lambda u^\sigma \partial_\sigma \phi^I\right)$$

$$P^{I\mu} = P^{\mu\nu}\partial_\nu\phi^I$$

- Conserved higher-form charge (lattice planes) [2]

$$Q^I[\Sigma_1] = \int_{\Sigma_1} \star J^{I(3)}$$



- Higher-form constitutive relations

$$u^\mu\partial_\mu\phi^I = -(\sigma^{-1})^{IJ}\partial_\mu\left(r_{JK}\,\partial^\mu\phi^K\right) \quad \Longrightarrow \quad T^{\mu\nu}[u^\mu, T, P^{I\mu}], \qquad J^{I\mu\nu\rho}[u^\mu, T, P^{I\mu}]$$

- Dual formulation

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^{I\mu\nu\rho} = 0$$

$$u^\mu \ \ (u^\mu u_\mu = -1), \qquad T, \qquad \zeta_{I\mu\nu} \ \ (u^\mu \zeta_{I\mu\nu} = 0)$$

[1] Grozdanov, Poovuttikul [1801.03199]
[2] Gaiotto, Kapustin, Seiberg, Willett [1412.5148]

DUAL IDEAL VISCOELASTICITY

- Ideal viscoelastic fluids:

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, P^{\mu\nu} - r_{IJ}\, \partial^\mu \phi^I \partial^\nu \phi^J$$

$$K_I = -\, \sigma_{IJ}\, u^\mu \partial_\mu \phi^J$$

$$dP = sdT + \frac{1}{2} r_{IJ} dh^{IJ}, \qquad \epsilon = sT - P$$

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\, P^{\mu\nu} - q^{IJ}\, \zeta_I^{\mu}{}_{\lambda} \zeta_J^{\nu\lambda}$$

$$J^{I\mu\nu\rho} = -\, 3 q^{IJ} u^{[\mu} \zeta_J^{\nu\rho]}$$

$$dp = sdT + \frac{1}{2} q^{IJ} d\gamma_{IJ}, \qquad \epsilon = sT + q^{IJ}\gamma_{IJ} - p$$

- Map:

$$\gamma_{IJ} = \frac{1}{2} \zeta_{I\mu\nu} \zeta_J^{\mu\nu}$$

$$\partial^\mu \phi^I = \frac{1}{2} q^{IJ} \epsilon^{\mu\nu\rho\sigma} u_\nu \zeta_{J\rho\sigma} + \ldots, \qquad h^{IJ} = q^{IK} q^{JL} \gamma_{KL} + \ldots$$

$$p = P - r_{IJ} h^{IJ} + \ldots, \qquad q^{IJ} = -\, (r^{-1})^{IJ}$$

The map can be similarly extended to higher derivative orders.

- Viscoelasticity is understood as hydrodynamics with 3 partially-broken 3-form symmetries.

CONFORMAL VISCOELASTIC FLUIDS

- Conformality:

$$T^{\mu}{}_{\mu} = 0 \implies \qquad \epsilon - 3P = -r_{IJ}h^{IJ}, \qquad h^{IJ}\eta_{IJKL} = \eta_{IJKL}h^{KL} = 0$$

- Linear viscoelasticity:

$$\epsilon_f - 3P_f = 3P_\ell, \qquad \epsilon_\ell - 3P_\ell = -3B, \qquad \zeta = 0$$

$$w = Ts + P_\ell$$
$$\epsilon_f = TP'_f - P_f$$
$$\epsilon_\ell = TP'_\ell - P_\ell$$

- Modes:

$$v_\parallel^2 = \frac{1}{3} + \frac{4}{3}\frac{G}{w}, \qquad \Gamma_\parallel = \frac{4}{3}\frac{T^2s^2}{\sigma w^2}\frac{4G^2}{w+4G} + \frac{4}{3}\frac{\eta}{w}$$

$$v_\perp^2 = \frac{G}{w}, \qquad \Gamma_\perp = \frac{T^2s^2G}{\sigma w^2} + \frac{\eta}{w}$$

$$D_\parallel = \frac{s^2T^2}{3\sigma(w+\epsilon_\ell)}\frac{4G - \epsilon_\ell}{w+4G}$$

Due to lattice pressure, there is a crystal diffusion mode even when $G = 0$

VISCOELASTIC HOLOGRAPHY

- Bulk theory: Einstein-Hilbert with 3 scalars Φ^I [1].
- Neumann boundary conditions for Φ^I to describe spontaneous symmetry breaking.
- Bulk action:

$$S = \frac{1}{16\pi G_N} \int d^5x \sqrt{-G} \left(R + 12 - 2V(H^{IJ}) \right) + counter\ terms \qquad H^{IJ} = G^{ab}\partial_a \Phi^I \partial_b \Phi^J$$

- X^N models: $V(H^{IJ}) = X^N$ $\qquad X = \frac{1}{2}\delta_{IJ}H^{IJ}$

$$ds^2 = 2dtdr + r^2 \left(-f(r)dt^2 + \delta_{IJ}dx^I dx^J \right), \qquad \Phi^I = \frac{m}{\sqrt{3/2}} x^I$$

Boundary thermodynamics (N=1):

$$P_f = \frac{1}{16\pi G_N} \left(r_0^4 + \frac{1}{3} m^2 r_0^2 \right), \qquad P_\ell = -\frac{1}{8\pi G_N} \frac{1}{3} m^2 r_0^2, \qquad T = \frac{r_0}{2\pi} \left(2 - \frac{1}{3} \frac{m^2}{r_0^2} \right)$$

[1] Horowitz et al. [1204.0519+]; Blake et al. [1310.3832+]; Esposito et al. [1708.09391+]; Alberte et al. [1708.08477+]; Baggioli et al. [1805.06756+]; Amoretti et al. [1711.06610+]

VISCOELASTIC HOLOGRAPHY

- Polynomial models: $V(H^{IJ}) = X + \lambda X^3$

$$P_\ell = -\frac{m^2 r_0^2}{8\pi G_N}\left(\frac{1}{3} - \lambda\frac{m^4}{r_0^4}\right), \qquad P_\ell = 0 \implies m = \frac{r_0}{(3\lambda)^{1/4}}$$

$$P_f = \frac{r_0^4}{16\pi G_N}\left(1 - \frac{2}{9\sqrt{3\lambda}}\right), \qquad T = \frac{r_0}{\pi}\left(1 - \frac{2}{9\sqrt{3\lambda}}\right)$$

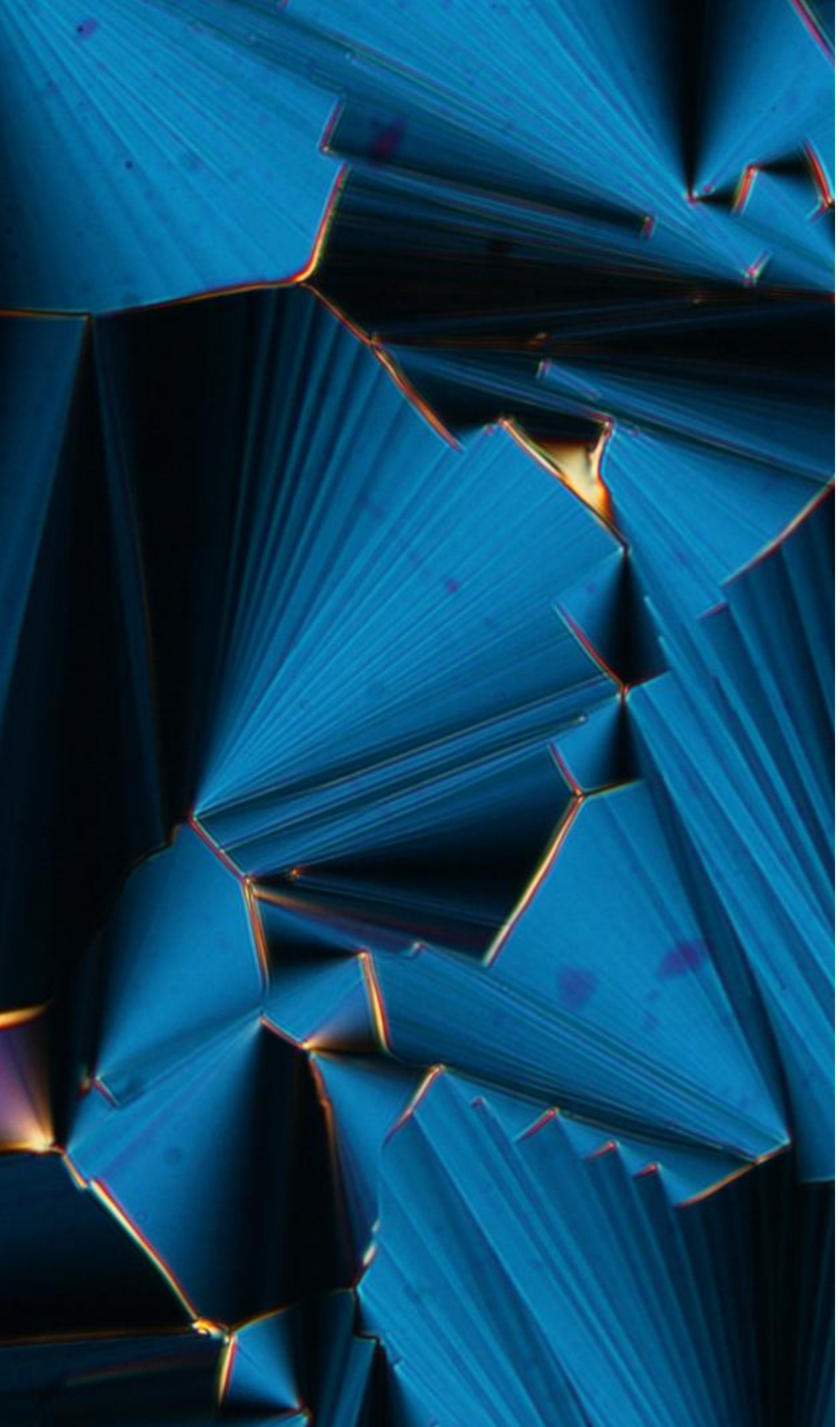
- Temperature-dependent equilibrium lattice configuration:

$$\phi^I \propto T x^I$$

Lattice collapses as temperature is taken to zero.

- Need to account for a temperature-dependent reference metric

$$\varepsilon_{IJ} = \frac{1}{2}\left(h_{IJ} - \hat{h}_{IJ}(T)\right)$$



OUTLOOK

- Zero lattice pressure holographic models with temperature-independent reference metric.
- Holographic models in dual formulation [1].
- Plastic materials: dynamical reference metric [2].
- Non-homogeneous crystals: gapped phonons [3].
- Liquid crystals: relaxed symmetry breaking structure [4].
- Wigner crystals: charged viscoelasticity [5].
- Topological defects: disclinations and dislocations in dual formulation.

[1] Grozdanov, Poovuttikul [1801.03199]; Armas, AJ [1908.01175]
[2] Fukuma, Sakatani [1104.1416]
[3] Musso et al. [1810.01799+]
[4] Martin, Parodi, Pershan; Jahnig, Schmidt 1972; Sonin, Vinen 1998
[5] Delacretaz et al. [1702.05104+]; Amoretti et al. [1711.06610+]


Viscoelastic hydrodynamics and

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Viscoelastic hydrodynamics and holography

Jay Armas (Amsterdam U.), Akash Jain (Victoria U.)

Aug 3, 2019 - 55 pages

e-Print: arXiv:1908.01175 [hep-th] | PDF

mySpires. Crystals | Elasticity | Higher Form Symmetries

Abstract (arXiv)
We formulate the theory of nonlinear viscoelastic hydrodynamics of anisotropic crystals in terms of dynamical Goldstone scalars of spontaneously broken translational symmetries, under the assumption of homogeneous lattices and absence of plastic deformations. We reformulate classical elasticity effective field theory using surface calculus in which the Goldstone scalars naturally define the position of higher-dimensional crystal cores, covering both elastic and smectic crystal phases. We systematically incorporate all dissipative effects in viscoelastic hydrodynamics at first order in a long-wavelength expansion and study the resulting rheology equations. In the process, we find the necessary conditions for equilibrium states of viscoelastic materials. In the linear regime and for isotropic crystals, the theory includes the description of Kelvin-Voigt materials. Furthermore, we provide an entirely equivalent description of viscoelastic hydrodynamics as a novel theory of higher-form superfluids in arbitrary dimensions where the Goldstone scalars of partially broken generalised global symmetries play an essential role. An exact map between the two formulations of viscoelastic hydrodynamics is given. Finally, we study holographic models dual to both these formulations and map them one-to-one via a careful analysis of boundary conditions. We propose a new simple holographic model of viscoelastic hydrodynamics by adopting an alternative quantisation for the scalar fields.

Note: v2: 55+1 pp, 1 fig, typos fixed, added references and appendix with comparison to previous literature
Keyword(s): INSPIRE: crystal: anisotropy | holography | spontaneous symmetry

THANK YOU

References

J Armas, AJ, [1908.01175, 1811.04913].

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