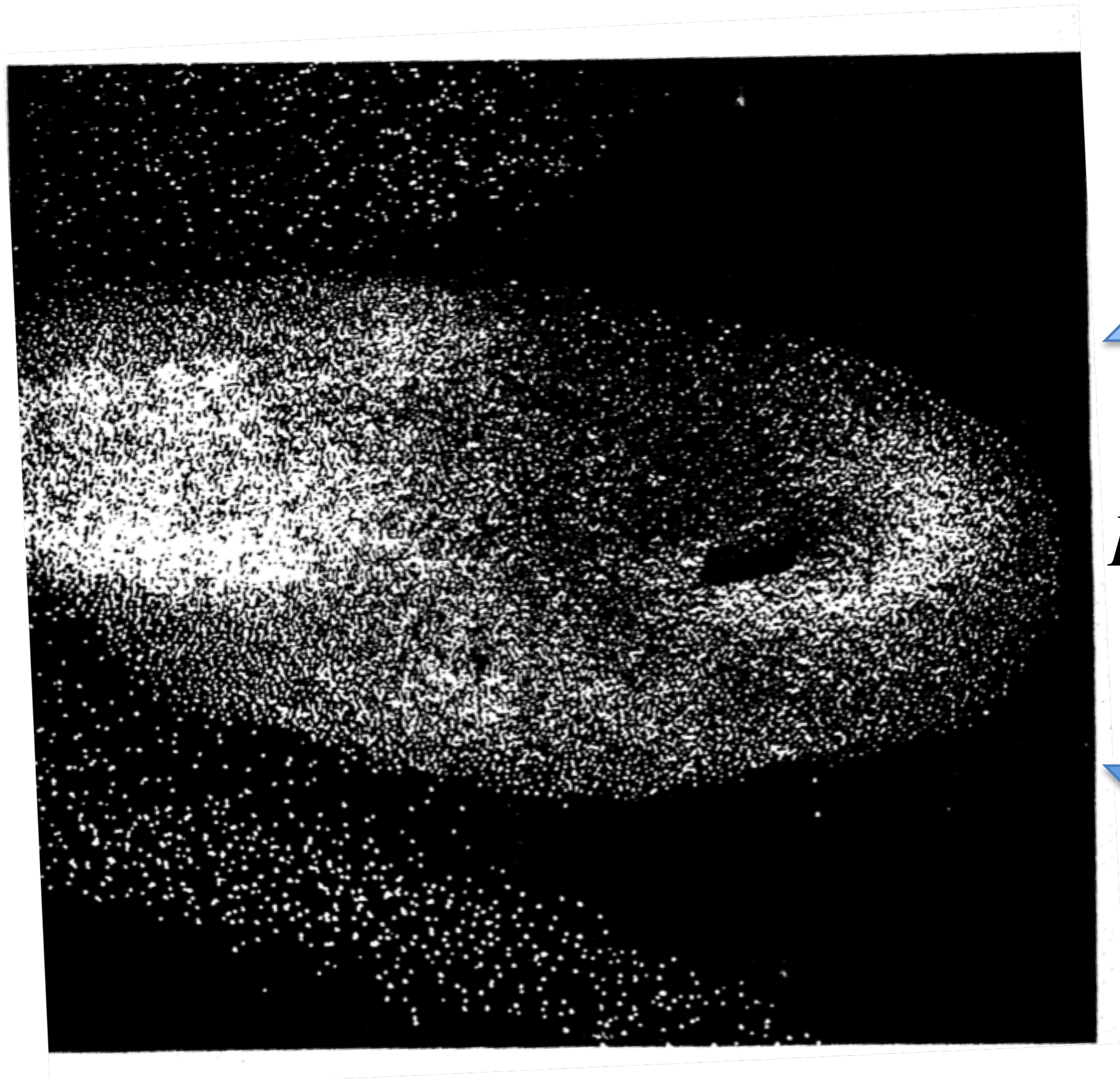
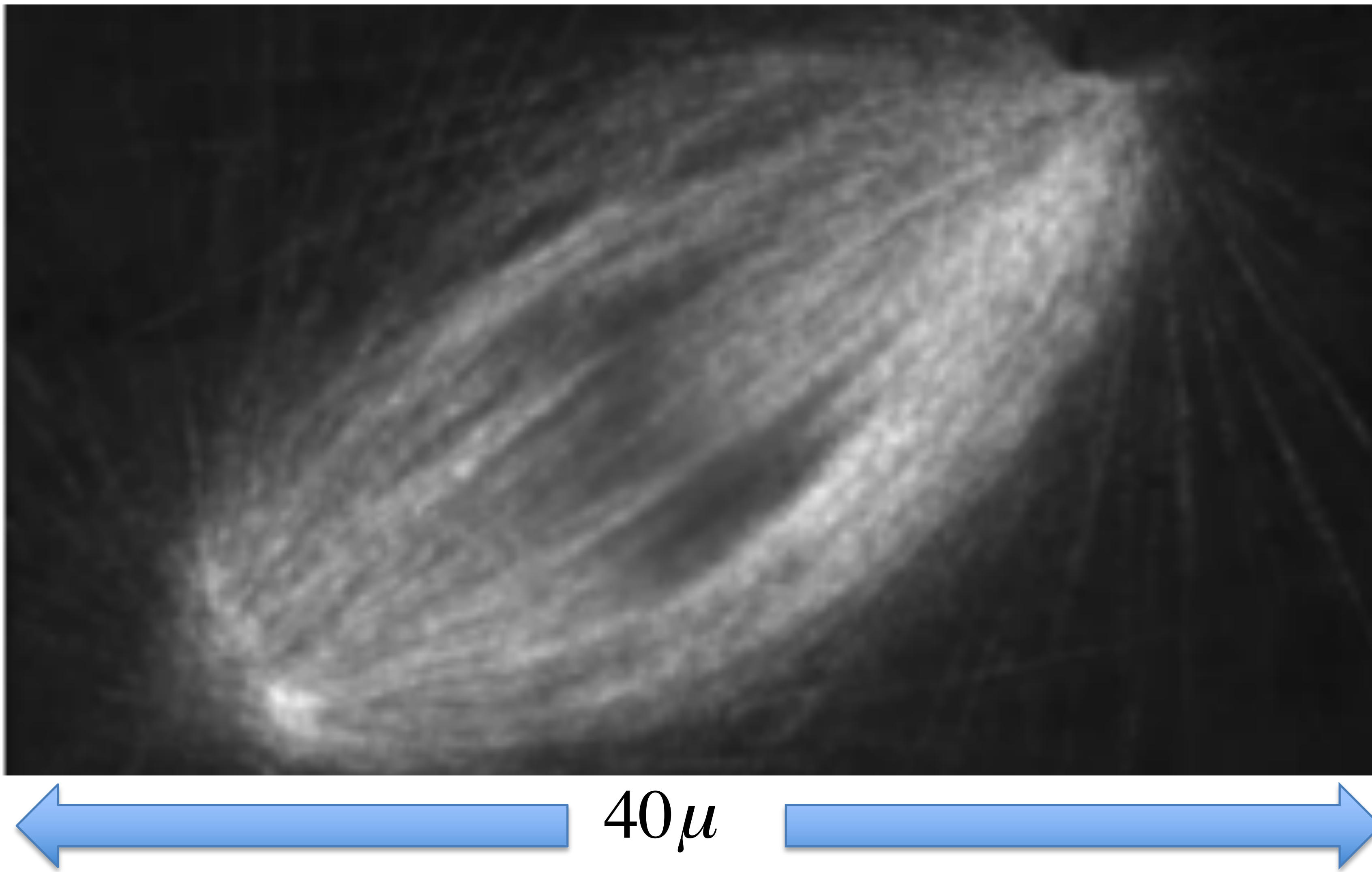
Birth, Death, and Flight: A Theory of Malthusian Flocks

An "Immortal" Flock: Flamingos in Africa (~10^5 birds)



$L \sim 1\,\text{km}$

A “Malthusian” flock: the Mitotic spindle



Outline

- I) Review of "Immortal" flocks (flocks with number conservation)
- (with Yu-hai Tu, IBM Watson)
- (J. Toner, Y.-h. Tu, and S. Ramaswamy, Ann. Phys. 318, 170 (2005))

- II) "Malthusian" flocks (number not conserved, due to "birth and death")

2d: (J. Toner, Phys. Rev. Lett. 108, 088102 (2012))

3d: Leiming Chen, Chiu Fan Lee, JT, in preparation

"The road to hell is paved with works in progress" (Philip Roth)

Immortal Flocks

I) Microscopic models (Vicsek)

Important points: Rotation Invariance
Locality

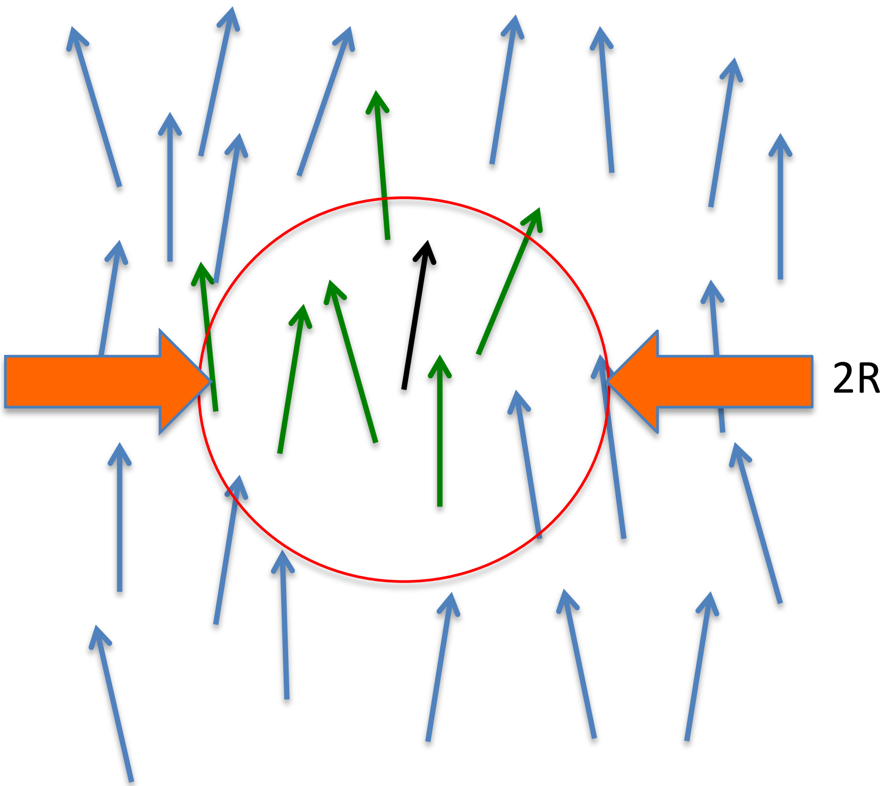
II) Mermin_Wagner Theorem: Are Birds smarter than nerds?

III) Continuum Theory: Analogy with Fluid Mechanics

IV) Predictions: How motion beats Mermin-Wagner

I) Microscopic Models

Vicsek algorithm:



$\vec{v}_i$ Velocity of Central bird "i"

$\vec{v}_j$ Velocity of Neighbors of central bird "j"

Update rule:

Errors

$$\vec{v}\ (t+1) = v_0 \left(\frac{\sum_j \vec{v}_j(t) + \vec{f}_i(t+1)}{\left| \sum_j \vec{v}_j(t) + \vec{f}_i(t+1) \right|} \right)$$

Speed of birds

Essential Features of Algorithm

- Only Local interactions: short ranged in space and time
- Ferromagnetic interactions (favor alignment)
- "Birds" keep moving ($\vec{v} \neq \vec{0}$) and making errors

Symmetries and conservation laws

Symmetries:

	Dynamics	Phase
Translation Invariance	YES	YES $<\rho(\vec{r},t)>\equiv\rho_0$ $=CONSTANT$
Rotation Invariance	YES	NO $<\vec{v}(\vec{r},t)>\equiv\vec{v}_0\neq\vec{0}$
Galilean Invariance	NO	NO

Conservation laws:

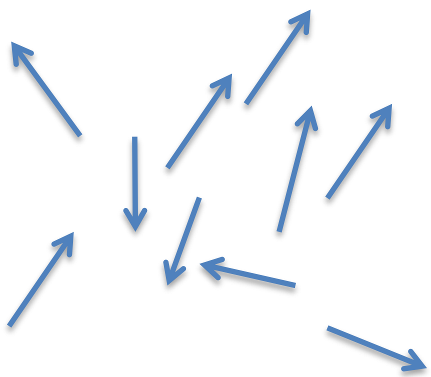
(For this part of the talk)

Bird number (density): Yes

Momentum: No (frictional substrate acts as a momentum sink)

("Dry" versus "wet")

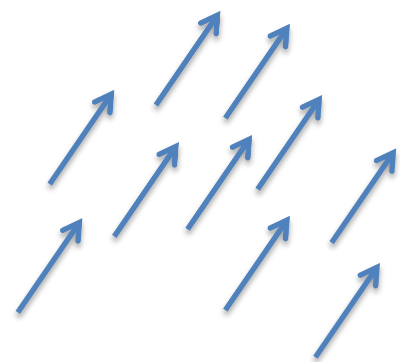
Dynamics produces order:



t=0

$$< \vec{v}(\vec{r},t) > = \vec{0}$$

Run algorithm
Many steps (t>>1)



t>>1

$$< \vec{v}(\vec{r},t) > = \vec{v}_0 \neq \vec{0}$$

However…..

This should be **Impossible!**

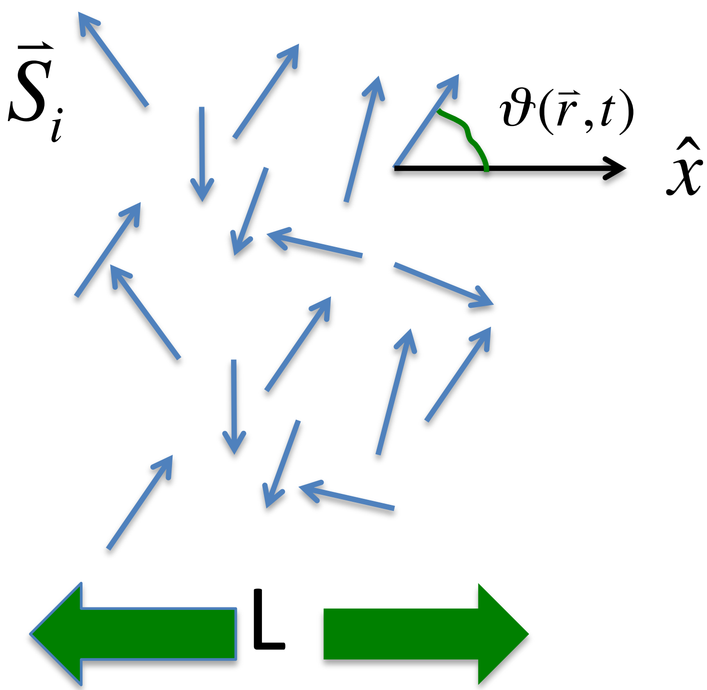
Why?

Violates Mermin-Wagner theorem

Are Birds smarter than nerds?

Mermin-Wagner theorem:
Pointers vs. Flockers

- APS pointers (“XY model”):



Error size

$$<\vartheta^2> = \Delta \ln(L) \xrightarrow[L\to\infty]{} \infty$$

⇒ No Long Ranged order

$$<\vec{S}(\vec{r},t)> = \vec{0}$$

Equilibrium result

Continuum Theory of Immortal Flocks

- Hard (impossible) to solve microscopic model with ~10^5 birds
- Harder to figure out what happens if you change model (universal vs system-specific)
- Historical analog: Fluid mechanics (Navier, Stokes, 1822):

  No theory of atoms and molecules

  No statistical physics

  No computers, ipad, ipod, etc

- So, how'd they do it?

Continuum Approach

Replace $\vec{r}_i(t) \longrightarrow$ Continuous fields:

$\rho(\vec{r},t)$: Coarse grained number density

$\vec{v}(\vec{r},t)$: Coarse grained velocity

Valid for: Length scales L >> interatomic distance

Time scales t >> collision time

Why these fields?

Why only these fields?

They're slow: lifetime $T(L \to \infty) \to \infty$

Why slow?

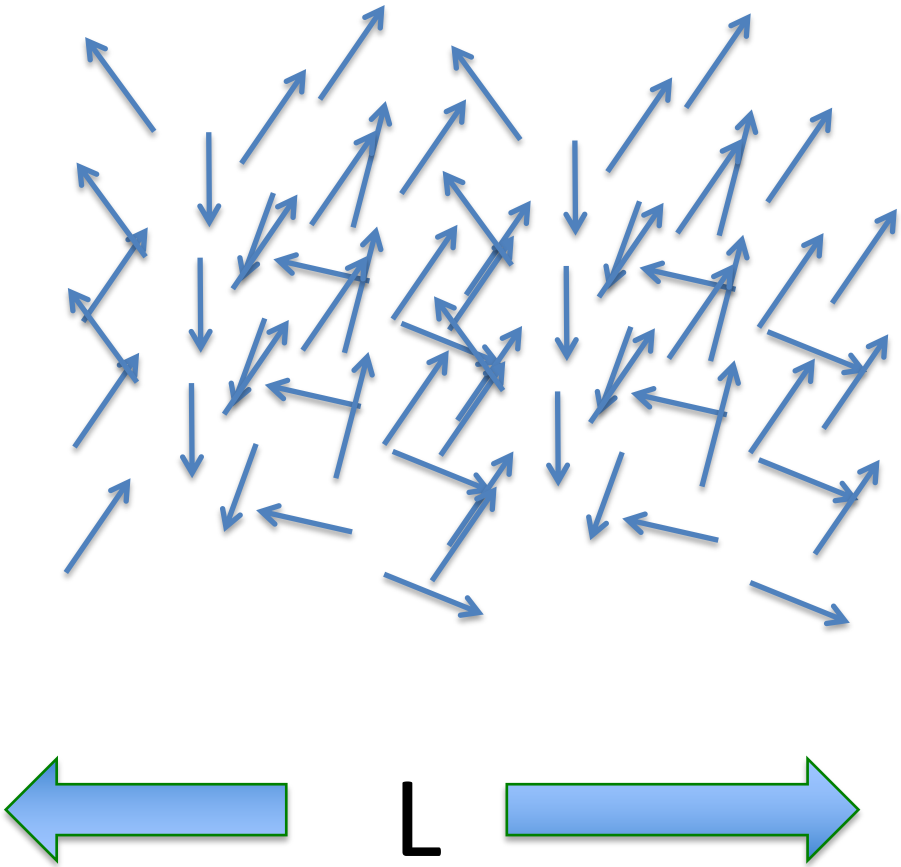
$\rho(\vec{r},t)$ Conserved For this part of talk!

$\vec{v}(\vec{r},t)$ Broken symmetry (Goldstone mode)

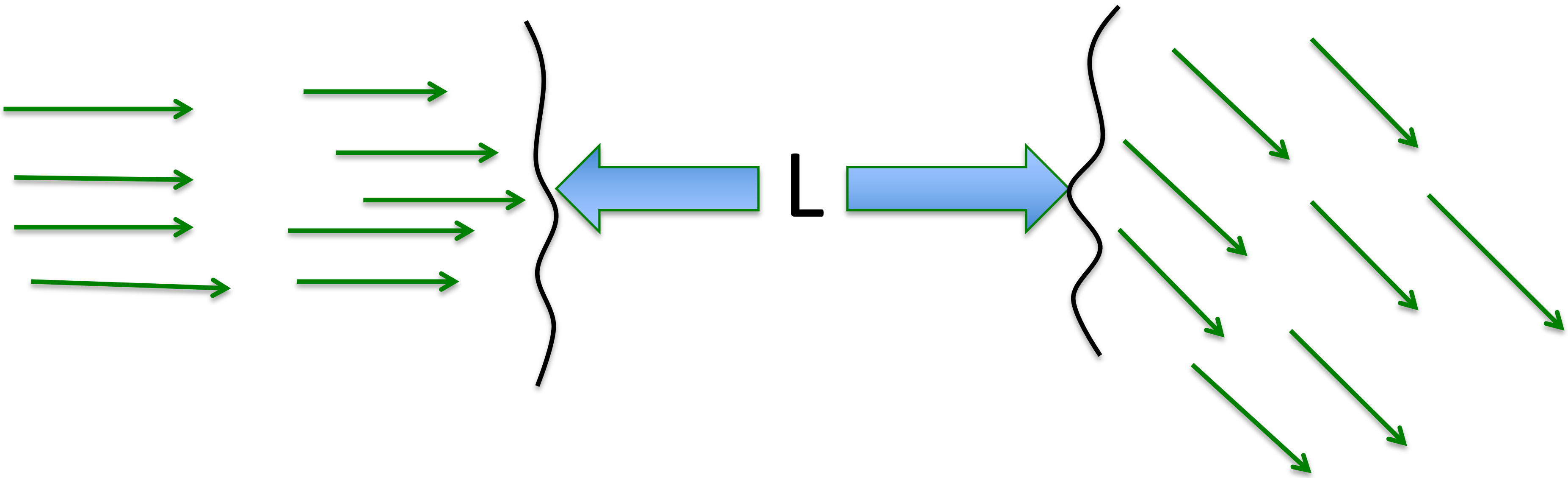
Must move particles a distance L to change ρ

Conserved=>slow:

=> $T(L \to \infty) \to \infty$



Broken symmetry=>slow (Goldstone's theorem)



Information must travel a distance L to relax this distortion (because the symmetry is spontaneously broken)=> $T(L\text{->}=>\infty)\ \text{->}\ \infty$

Why only these fields?

- Fast fields get enslaved to slow fields

I'll show this explicitly for flocks with birth and death, in which ρ becomes fast, because it's not conserved

$$\rho = \mathcal{F}(\{\mathbf{v}(\mathbf{r}, t)\})$$

For now, back to immortal flocks, ρ conserved

Equations of motion for $\rho(\vec{r},t), \vec{v}(\vec{r},t)$

Make 'em up!

Rules: -Lowest order in space, time derivatives
-Lowest order in fluctuations

$$\delta\rho(\vec{r},t) \equiv \rho(\vec{r},t) - \langle \rho(\vec{r},t) \rangle$$

$$\delta\vec{v}(\vec{r},t) \equiv \vec{v}(\vec{r},t) - \langle \vec{v}(\vec{r},t) \rangle$$

Respect Symmetries (for flocks, Rotation invariance)

Worked for fluids, should work for flocks

The Navier-Stokes Equations

Velocity EOM

"naïve acceleration" "convective derivative" "Shear viscosity" "Bulk viscosity"

$$\rho(\partial_t \vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v}) = -\vec{\nabla}P(\rho) + \eta_s \nabla^2 \vec{v} + \eta_B \vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + \vec{f}$$

"pressure"

"noise"
(models thermal
Fluctuations)

Density EOM

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

"Continuity equation"
(particle number conserved)

Our (Yu-hai Tu and JT) idea: same approach, different symmetry

- No Galilean invariance (birds move through a Special “rest frame” (e.g., air, water, surface of Serengeti. Etc....))

Hydrodynamic equations for Immortal Flocks:

New terms (forbidden in NS equations due to Galilean invariance)

Velocity EOM:

("convective Derivative")

Other "2 v grad v" terms

"move faster," Slowpoke

"but not TOO Fast!"

$$\partial_t \vec{v} + \lambda_1 (\vec{v}\cdot\vec{\nabla})\vec{v} + \lambda_2 \vec{v}(\vec{\nabla}\cdot\vec{v}) + \lambda_3(\vec{\nabla}|\vec{v}|^2) = \alpha\vec{v} - \beta|\vec{v}|^2\vec{v}$$

$$-\vec{\nabla}P(\rho) - \vec{v}(\vec{v}\cdot\vec{\nabla}P_2(\rho)) + D_B\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + D_T\nabla^2\vec{v} + D_2(\vec{v}\cdot\vec{\nabla})^2\vec{v} + \vec{f}$$

Anisotropic pressure

Anisotropic Viscosity

Noise (errors)

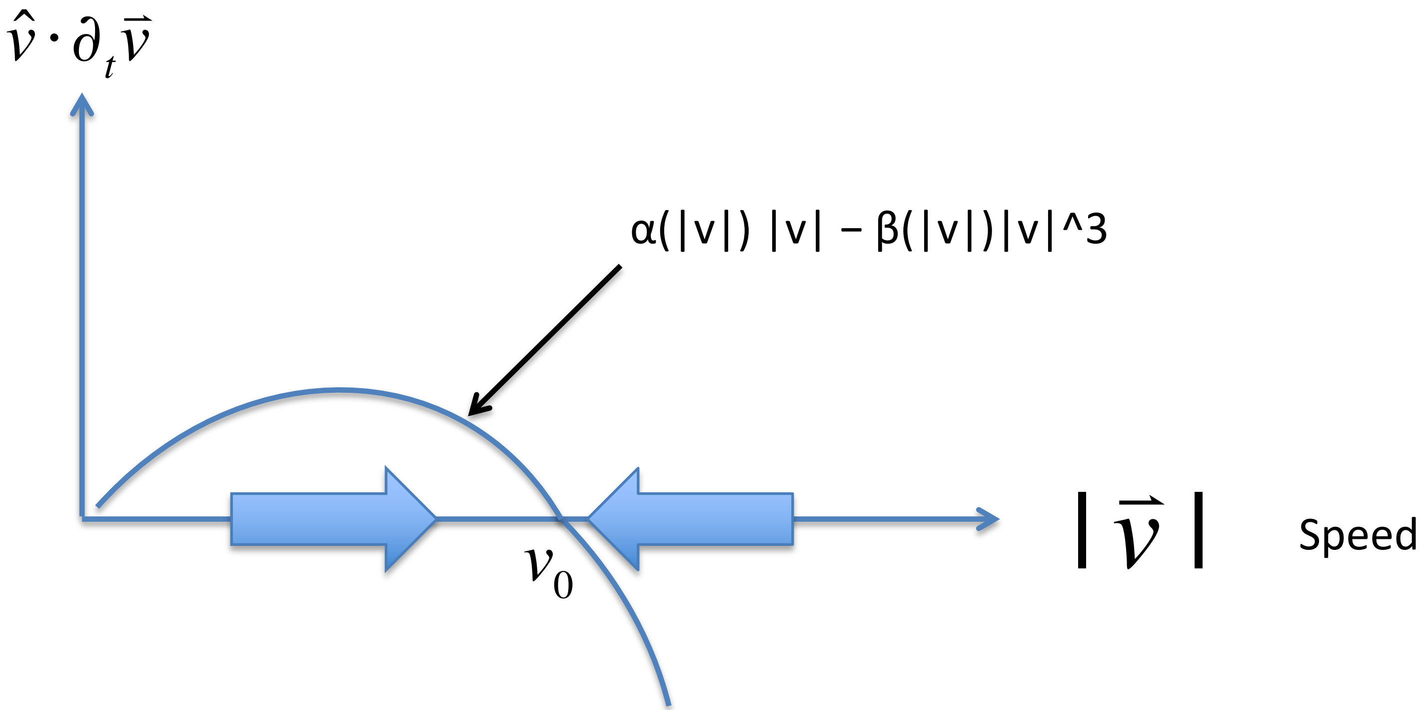
Density EOM:

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

Number conservation ("immortal" flock)

$\vec{f}(\vec{r},t)$ ： Langevin White Noise

Acceleration in direction of motion:



$$\Rightarrow <\vec{v}(\vec{r},t)> = v_0\hat{x}$$

Arbitrary direction (spontaneously broken symmetry)

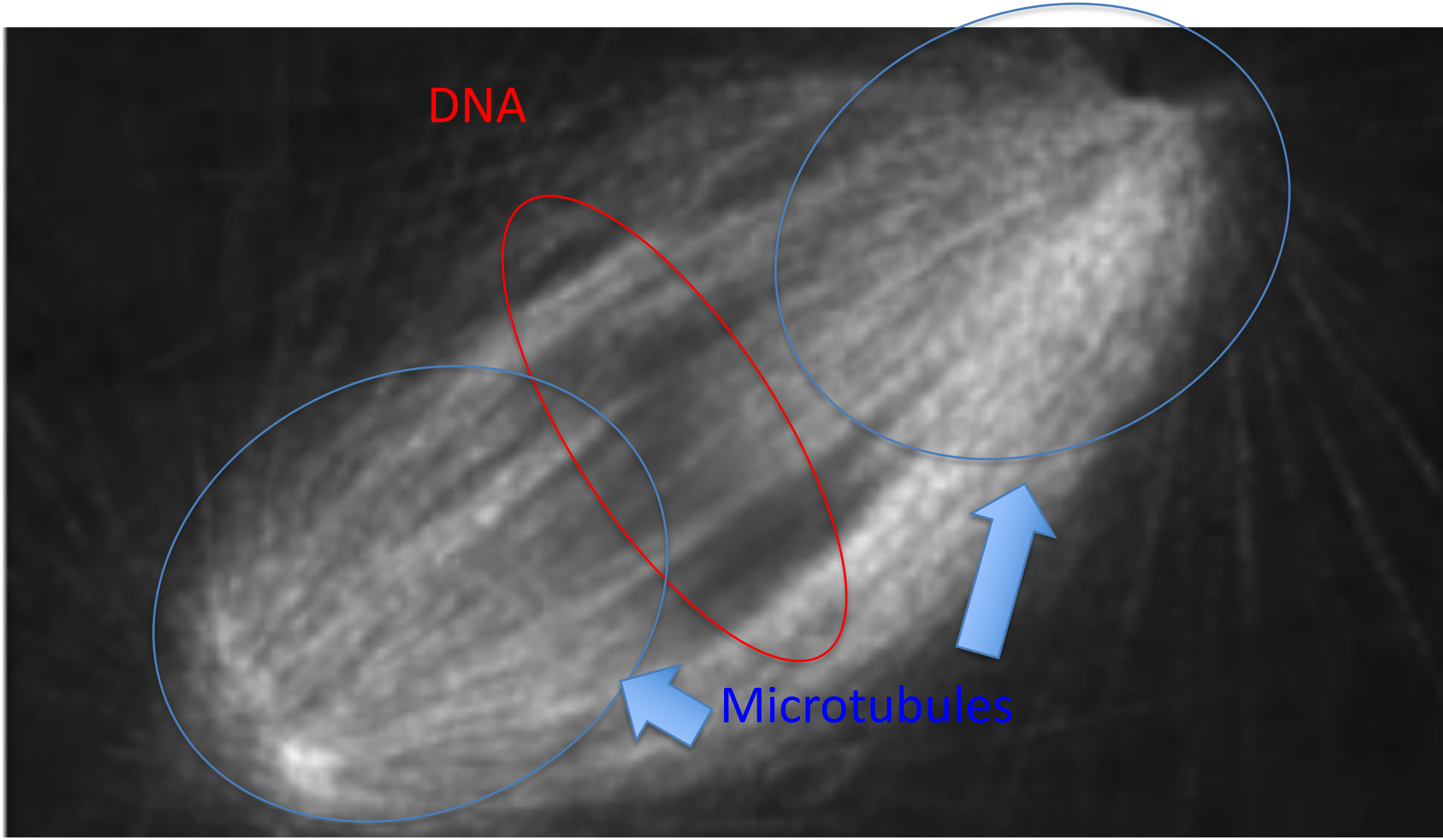
Predictions of hydrodynamic theory

- Sound waves
- Giant number fluctuations
- Anomalous Hydrodynamics for d<4.
- Long-ranged order in d=2

Malthusian Flocks: Flocks with Birth and Death during flight

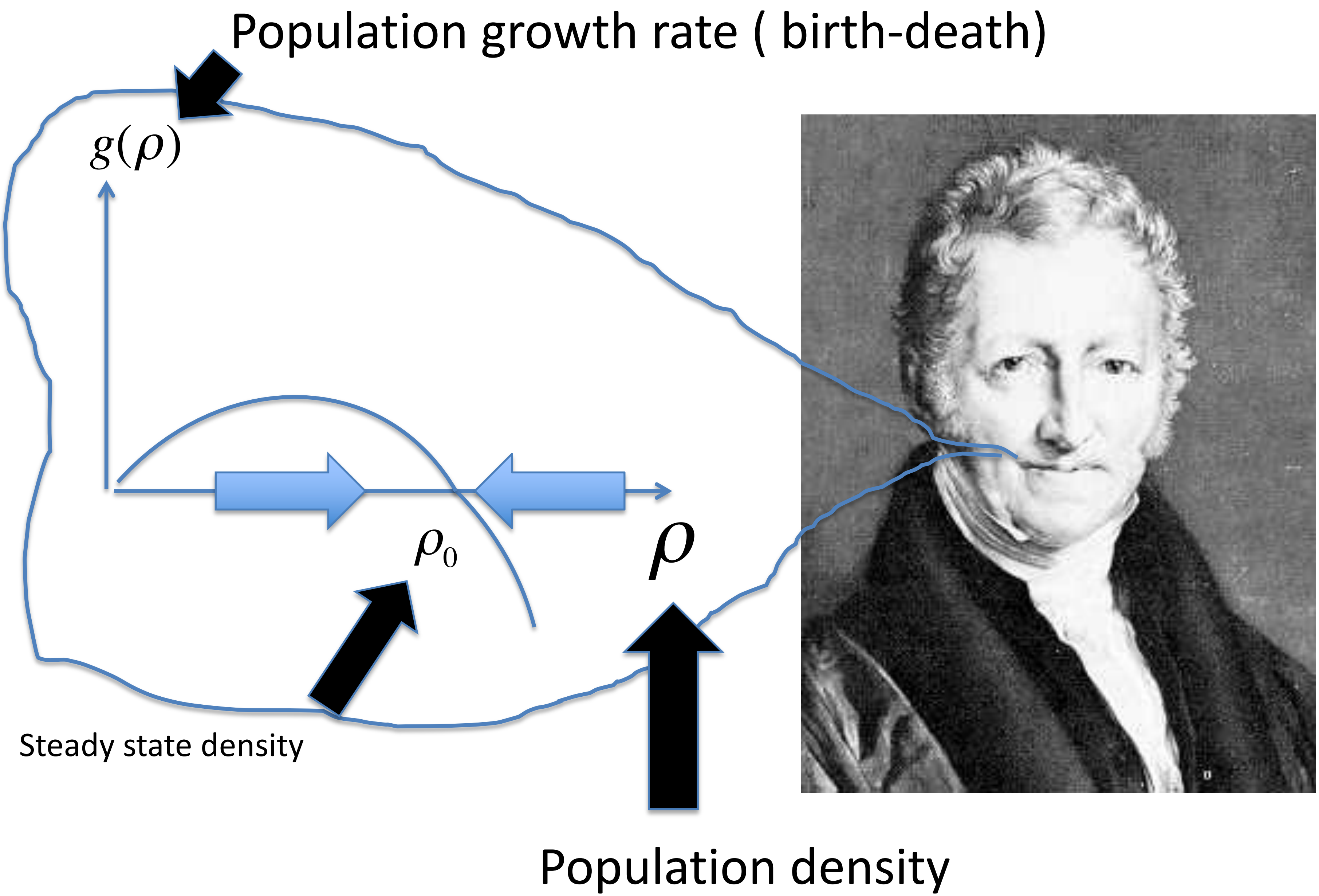
- Example: Mitotic spindle
- Homage to Malthus
- Hydrodynamic model
- Predictions:
- Still LRO ($< \vec{v} > \neq \vec{0}$) even in d=2.
- (And it outlives the birds!)
- No sound waves
- No Giant Number fluctuations
- But persistent number fluctuations

Mitotic spindle: Mechanism of cell reproduction



Thomas Malthus (1766-1834)

"The power of population is so superior to the power of the earth to produce subsistence for man, that premature death must in some shape or other visit the human race. The vices of mankind are active and able ministers of depopulation. They are the precursors in the great army of destruction, and often finish the dreadful work themselves. But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague advance in terrific array, and sweep off their thousands and tens of thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow levels the population with the food of the world".
—Malthus T.R. 1798. *An essay on the principle of population*. Chapter VII, p61[25]


Population growth rate (birth-death)
$g(\rho)$
ρ_0
ρ
Steady state density
Population density

Hydrodynamic equations for Malthusian flocks

Velocity EOM:

$$\partial_t \vec{v} + \lambda_1 (\vec{v} \cdot \vec{\nabla})\vec{v} + \lambda_2 \vec{v}(\vec{\nabla} \cdot \vec{v}) + \lambda_3 (\vec{\nabla} |\vec{v}|^2) = \alpha \vec{v} - \beta |\vec{v}|^2 \, \vec{v}$$

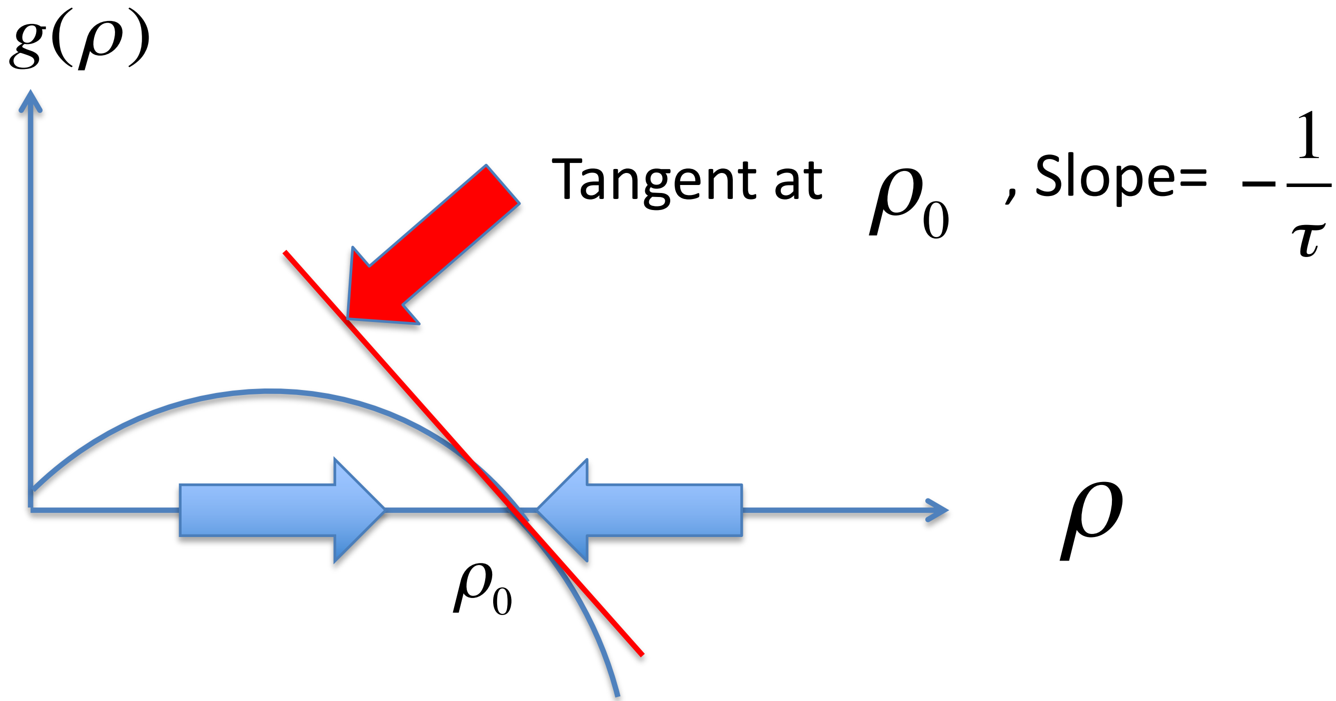
$$-\vec{\nabla} P(\rho) - \vec{v}(\vec{v} \cdot \vec{\nabla} P_2(\rho)) + D_B \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) + D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v} + \vec{f}$$

(Same as for immortal flocks)

Density EOM:

$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = g(\rho)$$

"Malthusian" form of $g(\rho)$



$\tau =$ Population relaxation time ~critter lifetime ~ 1 generation

Does population density relax to ρ_0 in time τ ?

No! To higher (lower) population set by immigration (emigration)

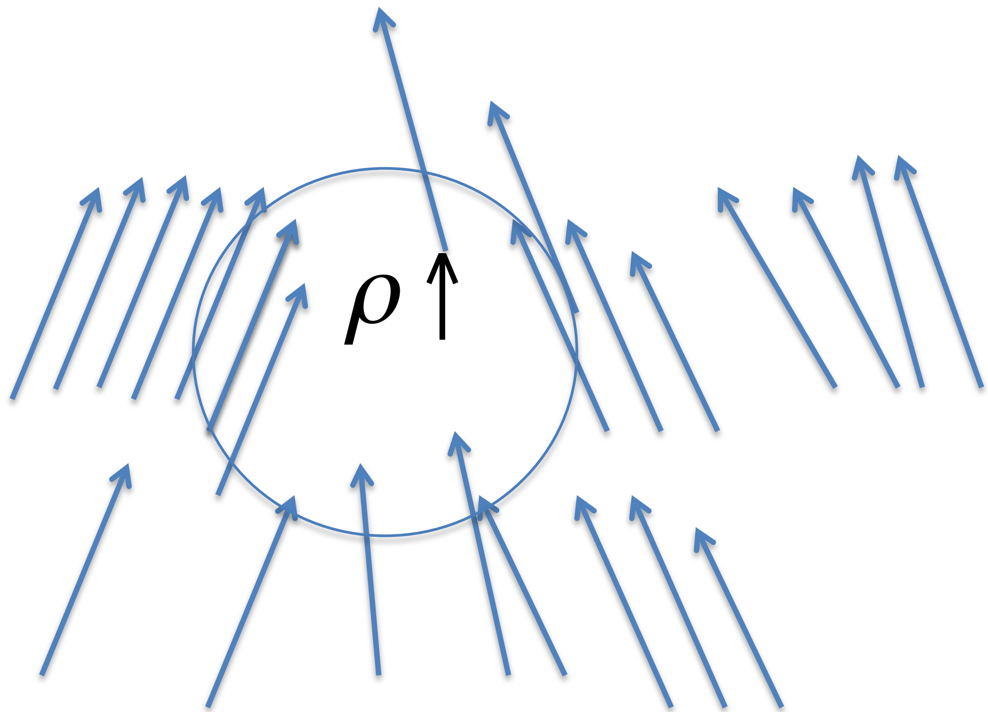
Negligible
(slow, hydrodynamic mode)

$$\partial_t \rho + \vec{\nabla} \cdot (\rho_0 \vec{v}) = g(\rho) \approx -(\rho - \rho_0)/\tau$$

immigration (emigration)

$$\Rightarrow \rho = \rho_0(1 - \tau \vec{\nabla} \bullet \vec{v}) \quad : \rho$$ Enslaved to $\vec{v}$

$$\vec{\nabla} \cdot \vec{v} < 0$$



This is what causes Giant Number Fluctuations in “Immortal” Flocks; here, it’s suppressed by increased death rate (Malthus was right!)

Substitute in v EOM, get closed EOM for v alone:

$$\partial_t \vec{v} + \lambda_1 (\vec{v} \cdot \vec{\nabla})\vec{v} + \lambda_2 \vec{v}(\vec{\nabla} \cdot \vec{v}) + \lambda_3 (\vec{\nabla} |\vec{v}|^2) = \alpha \vec{v} - \beta |\vec{v}|^2 \vec{v}$$

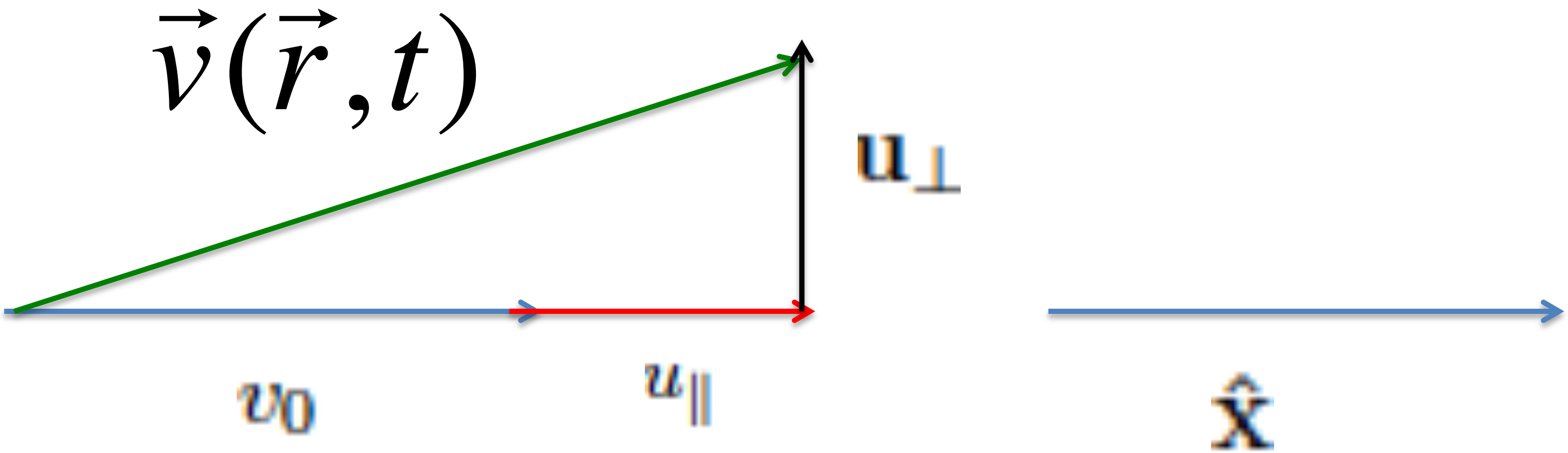
$$+ D_B' \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) + D_T \nabla^2 \vec{v} + D_2 (\vec{v} \cdot \vec{\nabla})^2 \vec{v} + \vec{f}$$

$$\rho \propto \vec{\nabla} \cdot \vec{v}$$

Expand in fluctuations:

$$\mathbf{v}(\mathbf{r},t) = (v_0 + u_{\|})\hat{\mathbf{x}} + \mathbf{u}_{\perp}(\mathbf{r},t)$$

Fast=>Eliminate



$\sim |\mathbf{r}_\perp|^{\chi - z}$

Gives theory for $\mathbf{u}_\perp$: $\sim |\mathbf{r}_\perp|^{(1-d-z-\zeta)/2}$

$$\partial_t \mathbf{u}_\perp + \lambda(\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp = \mu_1 \nabla_\perp^2 \mathbf{u}_\perp + \mu_2 \nabla_\perp(\nabla_\perp \cdot \mathbf{u}_\perp) + \mu_x \partial_x^2 \mathbf{u}_\perp + \mathbf{f}_\perp$$

Linear

$\sim |\mathbf{r}_\perp|^{\chi - 2}$

Linear

$\sim |\mathbf{r}_\perp|^{\chi - 2\zeta}$

Nonlinear

Power counting:

Dynamic exponent z:

$$\langle f_i(\mathbf{r},t) f_j(\mathbf{r}',t') \rangle = 2D\delta_{ij}\delta^d(\mathbf{r}-\mathbf{r}')\delta(t-t')$$
$$\sim \delta^{d-1}(\mathbf{r}_\perp - \mathbf{r}'_\perp)\delta(x-x')\delta(t-t')$$
$$\sim x^{-1}|\mathbf{r}_\perp|^{-(d-1)}t^{-1}$$

$$t \sim |\mathbf{r}_\perp|^z$$

$$x \sim |\mathbf{r}_\perp|^\zeta$$

$$\mathbf{f} \sim \sqrt{x^{-1}|\mathbf{r}_\perp|^{-(d-1)}t^{-1}} \sim |\mathbf{r}_\perp|^{(1-d-z-\zeta)/2}$$

$$\mathbf{u}_\perp \sim |\mathbf{r}_\perp|^\chi$$

Linear theory scaling laws:

Equate powers of $|\mathbf{r}_\perp|$ of linear terms:

$$\underbrace{\partial_t \mathbf{u}_\perp}_{\sim |\mathbf{r}_\perp|^{\chi - z}} + \lambda(\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp = \underbrace{\mu_1 \nabla_\perp^2 \mathbf{u}_\perp + \mu_2 \nabla_\perp(\nabla_\perp \cdot \mathbf{u}_\perp)}_{\sim |\mathbf{r}_\perp|^{\chi - 2}} + \underbrace{\mu_x \partial_x^2 \mathbf{u}_\perp}_{\sim |\mathbf{r}_\perp|^{\chi - 2\zeta}} + \underbrace{\mathbf{f}_\perp}_{\sim |\mathbf{r}_\perp|^{(1-d-z-\zeta)/2}}$$

$$\chi - z = \chi - 2 \ \Rightarrow z = 2$$

$$\chi - 2 = \chi - 2\zeta \ \Rightarrow \zeta = 1$$

$$(1 - d - z - \zeta)/2 = \chi - 2$$
$$\Rightarrow \chi = 2 + (1 - d - 2 - 1)/2 = (2 - d)/2$$

Note: χ =0 in d=2

$\mathbf{u}_\perp \sim \ln(|\mathbf{r}_\perp|) \to \infty \Rightarrow$ No LRO

(Mermin-Wagner Thm)

Can Malthusian Flock in d=2 have LRO? Yes! Nonlinearity to the rescue!

$\sim |\mathbf{r}_\perp|^{\chi - z}$

$$\partial_t \mathbf{u}_\perp + \lambda(\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp = \mu_1 \nabla_\perp^2 \mathbf{u}_\perp + \mu_2 \nabla_\perp(\nabla_\perp \cdot \mathbf{u}_\perp) + \mu_x \partial_x^2 \mathbf{u}_\perp + \mathbf{f}_\perp$$

$\sim |\mathbf{r}_\perp|^{2\chi - 1}$

$$\mathbf{u}_\perp \sim |\mathbf{r}_\perp|^{\chi}$$

$$\text{ratio} \sim |\mathbf{r}_\perp|^{\chi + z - 1} = |\mathbf{r}_\perp|^{(4-d)/2} \to \infty$$

$$z = 2$$

$$\zeta = 1$$

$$\chi = (2-d)/2$$

$\Longrightarrow$ Nonlinearity changes scaling for d<4!

So what *is* new true scaling for d<4? d=2 is simple:

JT, PRL 108, 088102 (2012)

$$(\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp \rightarrow u_y \partial_y u_y = \frac{1}{2}\partial_y(u_y^2)$$

Total y derivative!

$$\partial_t u_y + \frac{\lambda}{2}\partial_y(u_y^2) = (\mu_1 + \mu_2)\partial_y^2 u_y + \mu_x \partial_x^2 u_y + f_y$$

Nonlinear power counting:

$\sim |\mathbf{r}_\perp|^{\chi - z}$

$\sim |\mathbf{r}_\perp|^{(1-d-z-\zeta)/2}$

Neglect (dominated by NL term)

$$\partial_t u_y + \frac{\lambda}{2}\partial_y(u_y^2) = (\mu_1+\mu_2)\partial_y^2 u_y + \mu_x \partial_x^2 u_y + f_y$$

$\sim |\mathbf{r}_\perp|^{2\chi - 1}$

$\sim |\mathbf{r}_\perp|^{\chi - 2\zeta}$

3 linear equations,
3 unknowns: solution:

$$\chi - z = 2\chi - 1$$

$$\chi - z = \chi - 2\zeta \Rightarrow z = 2\zeta$$

$$(1 - d - z - \zeta)/2 = \chi - z$$

$$z(d=2) = \frac{2(d+1)}{5} = \frac{6}{5}$$

$$\zeta(d=2) = \frac{d+1}{5} = \frac{3}{5}$$

$$\chi(d=2) = \frac{3-2d}{5} = -\frac{1}{5}$$

χ=-1/5<0=> long-ranged order!

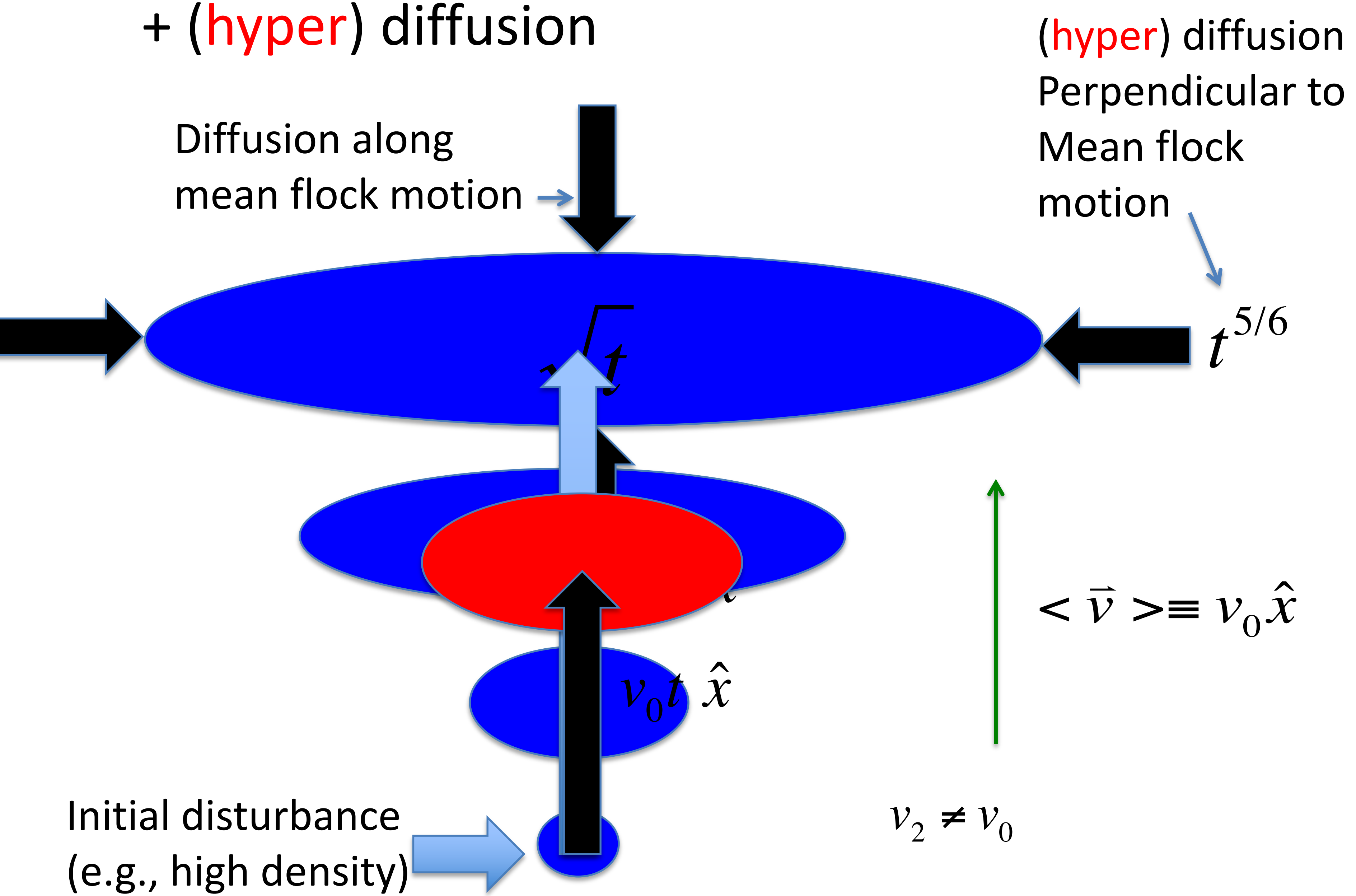
$$< \vec{v} > \neq \vec{0}$$

And it outlives the birds!

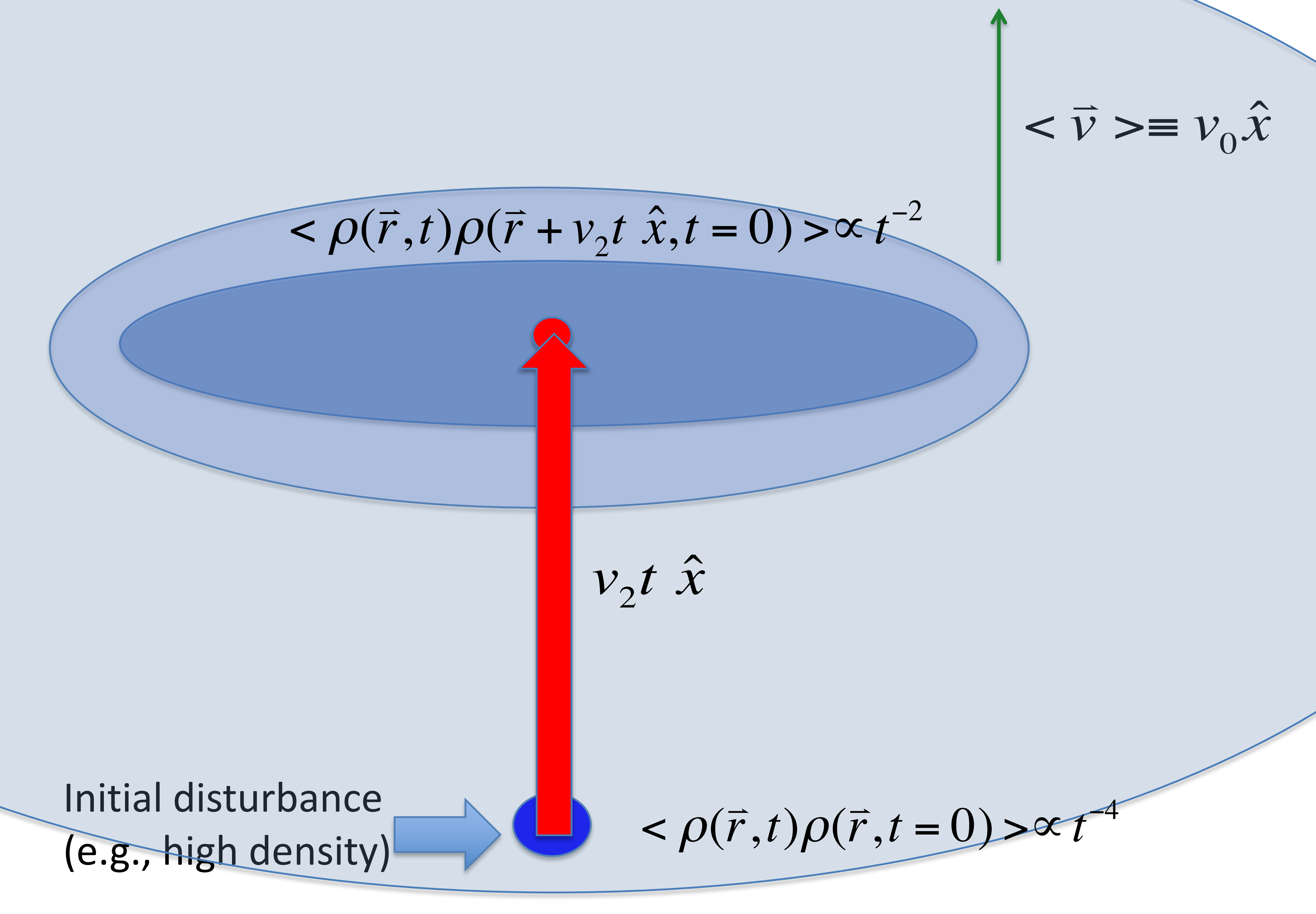
Persistence time T of $< \vec{v}(\vec{r},t) >$ $>> \tau$ Lifetime Of one bird

$T \propto N$ (number of birds) $>>$ τ Independent of N

No sound modes; instead, convention + (hyper) diffusion



Persistent number fluctuations



What about d=3?

With Chiu Fan Lee, Imperial college, London

Leiming Chen, China University of Mining and Technology

$(\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp$ Is *not* a total derivative

=> No simple power counting argument

=> Must do full blown dynamical RG $\epsilon = 4 - d$ expansion

We thought it would be easy:

We were WRONG!

Nonlinear coupling:

$$\frac{1}{g_1}\frac{\mathrm{d}g_1}{\mathrm{d}\ell} = 2 + g_1\left(G_U - \frac{5}{2}G_{\mu}\right) \equiv \ldots \qquad g_1 \equiv \frac{D\lambda^2}{\sqrt{\mu_x \mu_1^5}}\frac{S_{d-1}}{(2\pi)^{d-1}}\Lambda^{d-4} \quad , \quad g_2 \equiv \frac{\mu_2}{\mu_1}$$

$$\frac{1}{g_2}\frac{\mathrm{d}g_2}{\mathrm{d}\ell} = g_1(G_{\mu_2} - G_{\mu_1}) \equiv g_1 G_{g_2}(g_2)\,,$$

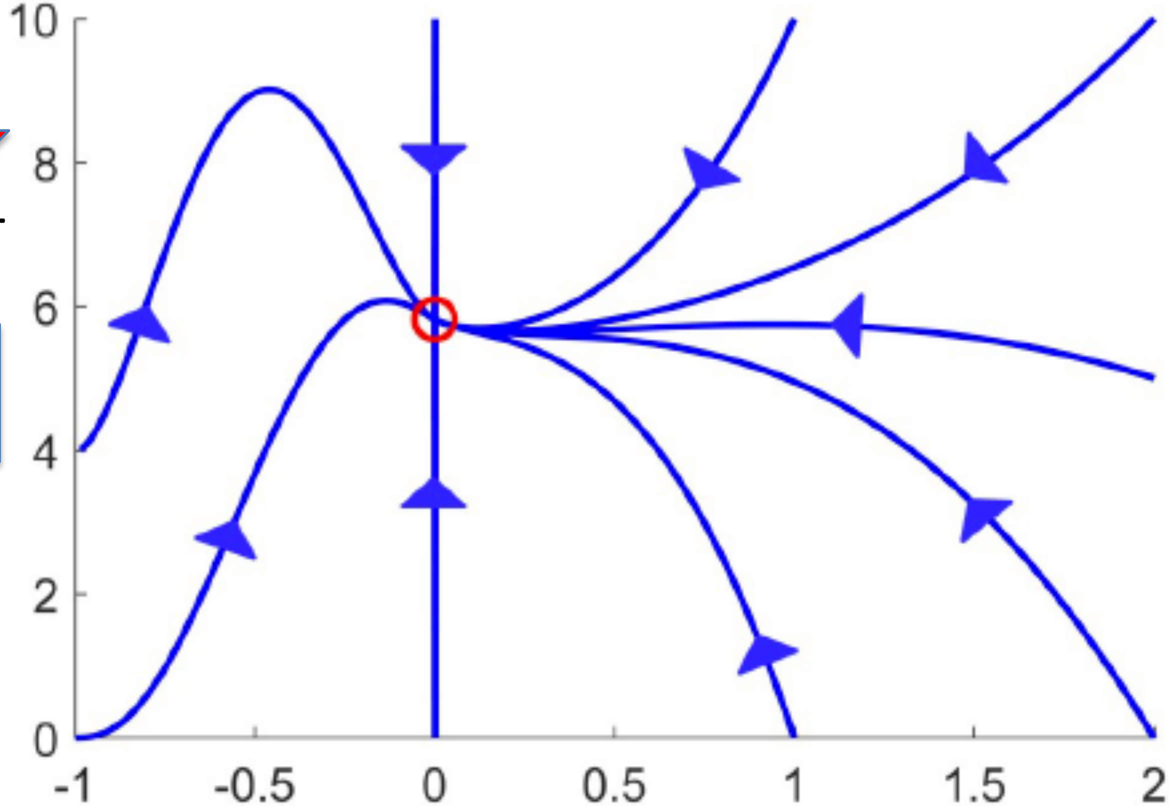
$$\begin{aligned} G_{g_1}(g_2) = {} & \frac{(-10d^2+30d-15)}{32(d^2-1)} + \frac{(d^2-d+8)}{2(d^2-1)g_2^2} - \frac{(2d^2+3d+11)\sqrt{2}}{(d^2-1)g_2^2(g_2+2)^{3/2}} + \frac{(d+3)}{2(d-1)g_2^2\sqrt{g_2+1}} + \frac{(15-5d)}{2(d^2-1)g_2} \\ & - \frac{\sqrt{2}(4d^2-9d+97)}{4(d^2-1)g_2(g_2+2)^{3/2}} + \frac{(5d-45)}{2\sqrt{2}(d^2-1)(g_2+2)^{3/2}} + \frac{5}{4(d-1)g_2\sqrt{g_2+1}} \end{aligned}$$

$$\begin{aligned} G_{g_2}(g_2) = {} & \left(\frac{2}{d^2-1}\right)\left(\frac{(1-3d)\sqrt{2}}{g_2^3(g_2+2)^{3/2}} + \frac{(d-1)}{g_2^3} + \frac{(d+1)}{2g_2^3\sqrt{g_2+1}} + \frac{(1-19d)}{2\sqrt{2}g_2^2(g_2+2)^{3/2}} + \frac{(d^2-4d+3)}{2\sqrt{2}g_2^2\sqrt{g_2+2}} - \frac{(d^2-7d+4)}{4g_2^2}\right. \\ & + \frac{3(d+1)}{4g_2^2\sqrt{g_2+1}} - \frac{3\sqrt{2}}{2(g_2+2)^{3/2}} - \frac{(9+7d)}{2\sqrt{2}g_2(g_2+2)^{3/2}} + \frac{(2d^2-25d+59)}{32g_2} + \frac{d+1}{64g_2(g_2+1)^{3/2}} + \frac{(d+1)}{4g_2\sqrt{g_2+1}} \\ & \left. + \frac{(d-3)}{2\sqrt{2}g_2\sqrt{g_2+2}} - \frac{(2d^2-6d+3)}{32}\right) \end{aligned}$$

RG flows:

Non-linear coupling:

g_1



g_2

$$\frac{\text{bulk viscosity} - \text{shear viscosity}}{\text{shear viscosity}}$$

Results:

$$
\begin{aligned}
z &= 2 - \frac{6\epsilon}{11} + \mathcal{O}(\epsilon^2) \\
\chi &= -1 + \frac{6\epsilon}{11} + \mathcal{O}(\epsilon^2) \\
\zeta &= 1 - \frac{3\epsilon}{11} + \mathcal{O}(\epsilon^2) \, .
\end{aligned}
$$

$$z = \frac{28}{19} \approx 1.47 \, ,$$

$$\zeta = \frac{14}{19} \approx 0.74 \, ,$$

$$\chi = -\frac{9}{19} \approx -0.47 \, .$$

Summary:

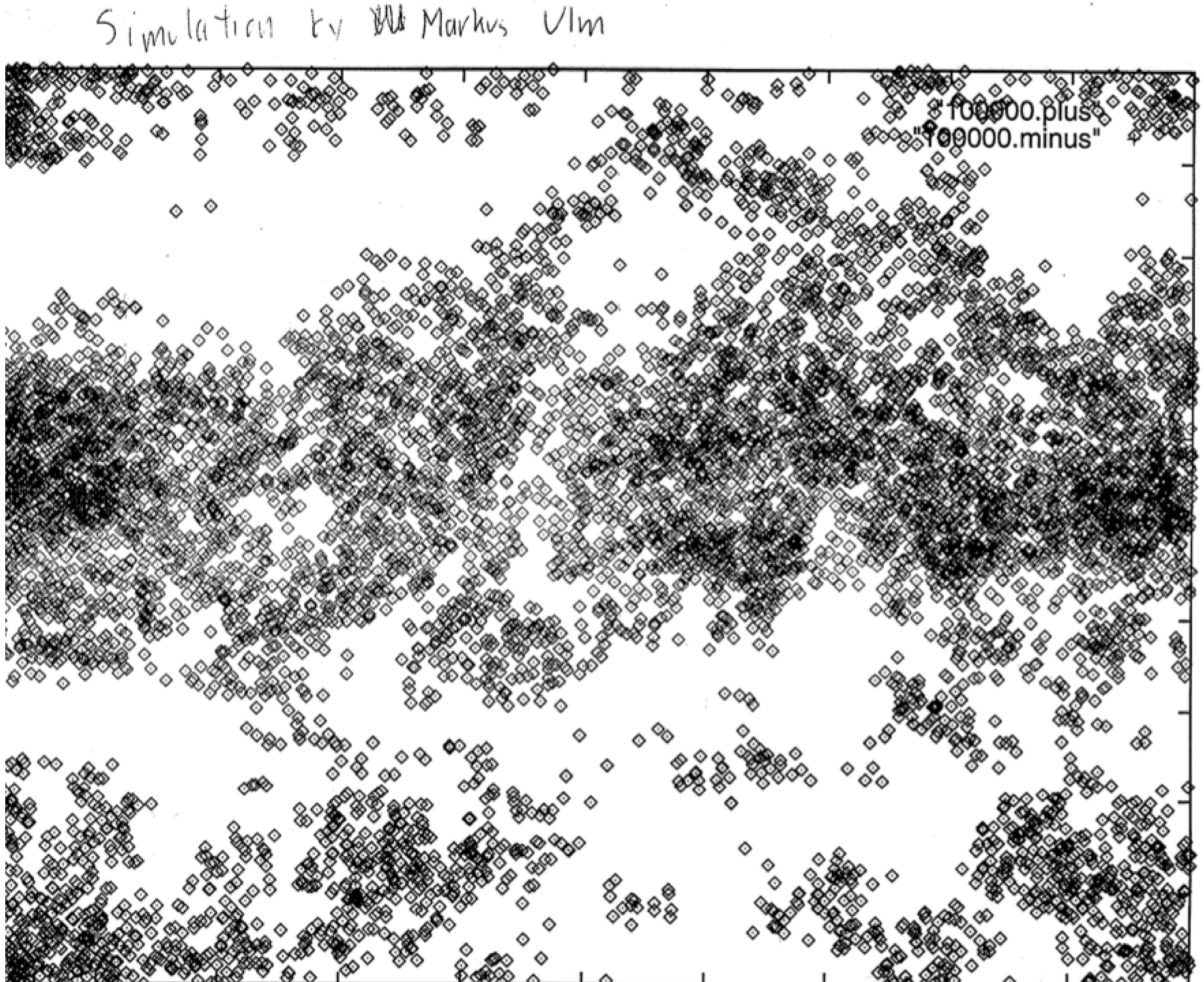
- Flocks illustrate fundamental Hydrodynamic principles (symmetries, conservation laws)
- Birth and death turn off number conservation, which changes long length, time scale behavior of flocks radically (no sound, no GNF)
- But can still understand using hydrodynamic approach

Thanks for your attention!

And don’t have any more !@#$%^&*

Kids!!!!!!

Giant number fluctuations



Anomalous Hydrodynamics

Hydrodynamics With noise	$\neq$	Hydrodynamics Without noise

Occurs for any $d<4$ (i.e., $d=2$ and $d=3$)

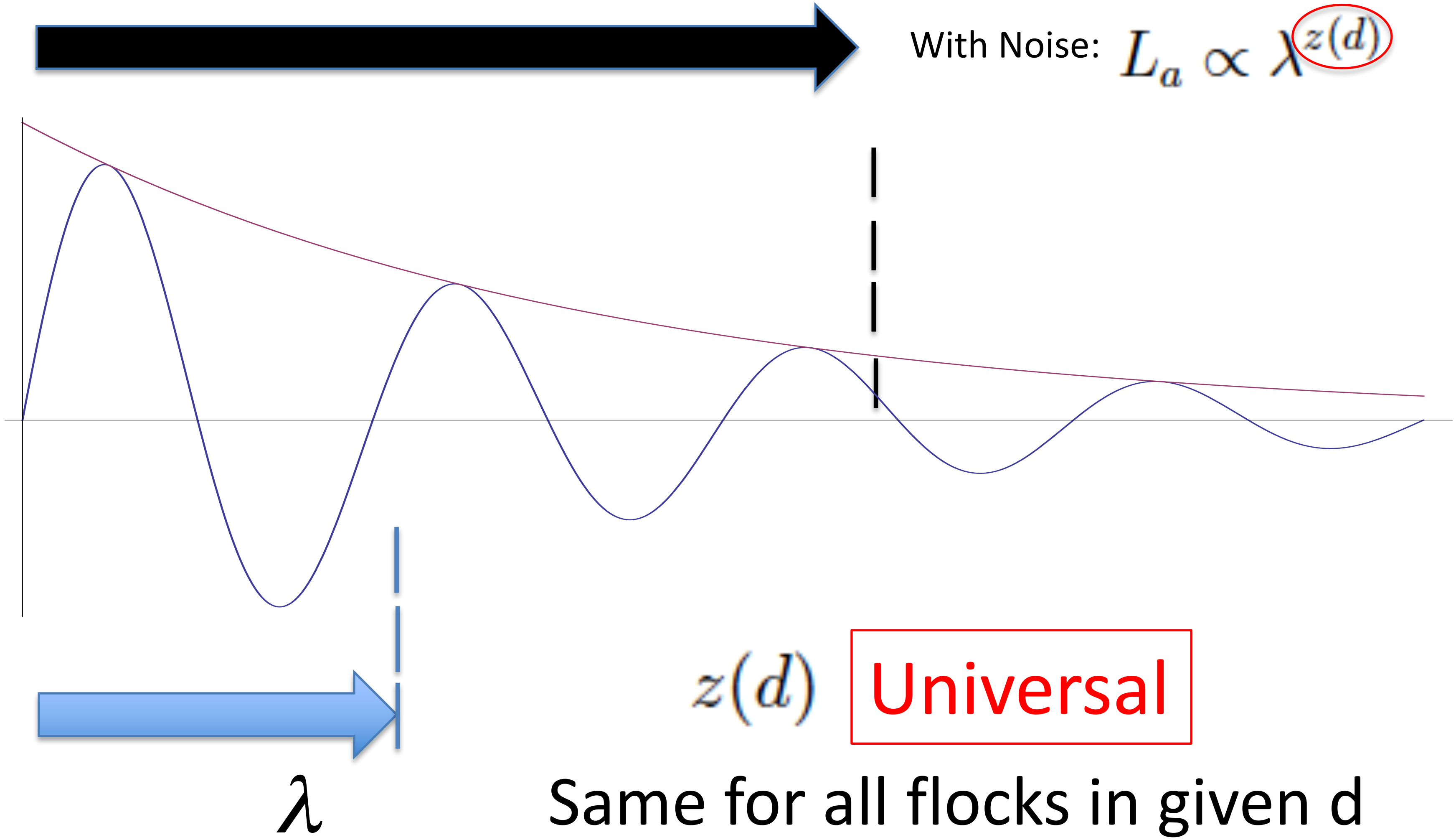
In $d=2$: Only way to stabilize LRO!

Example: Sound damping:

Attenuation length L_a

Without Noise: $L_a \propto \lambda^{2}$

With Noise: $L_a \propto \lambda^{z(d)}$



$z(d)$ **Universal**

Same for all flocks in given d

But d-dependent, and z(d<4)<2

Why does this happen?

Fluctuations (waves) interact due to

Convective nonlinearity

and other nonlinearities from density:

$$\partial_t \vec{v} + \lambda_1(\vec{v}\cdot\vec{\nabla})\vec{v} + \lambda_2\vec{v}(\vec{\nabla}\cdot\vec{v}) + \lambda_3(\vec{\nabla}|\vec{v}|^2) = \alpha\vec{v} - \beta|\vec{v}|^2\,\vec{v}$$

$$-\vec{\nabla}P(\rho) - \vec{v}(\vec{v}\cdot\vec{\nabla}P_2(\rho)) + D_B\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + D_T\nabla^2\vec{v} + D_2(\vec{v}\cdot\vec{\nabla})^2\vec{v} + \vec{f}$$

Calculating z(d):

Nightmare!

In principle, straightforward dynamical RG calculation

But: Calculation only valid near d=4

Worse: Even that calculation is incredibly hard!

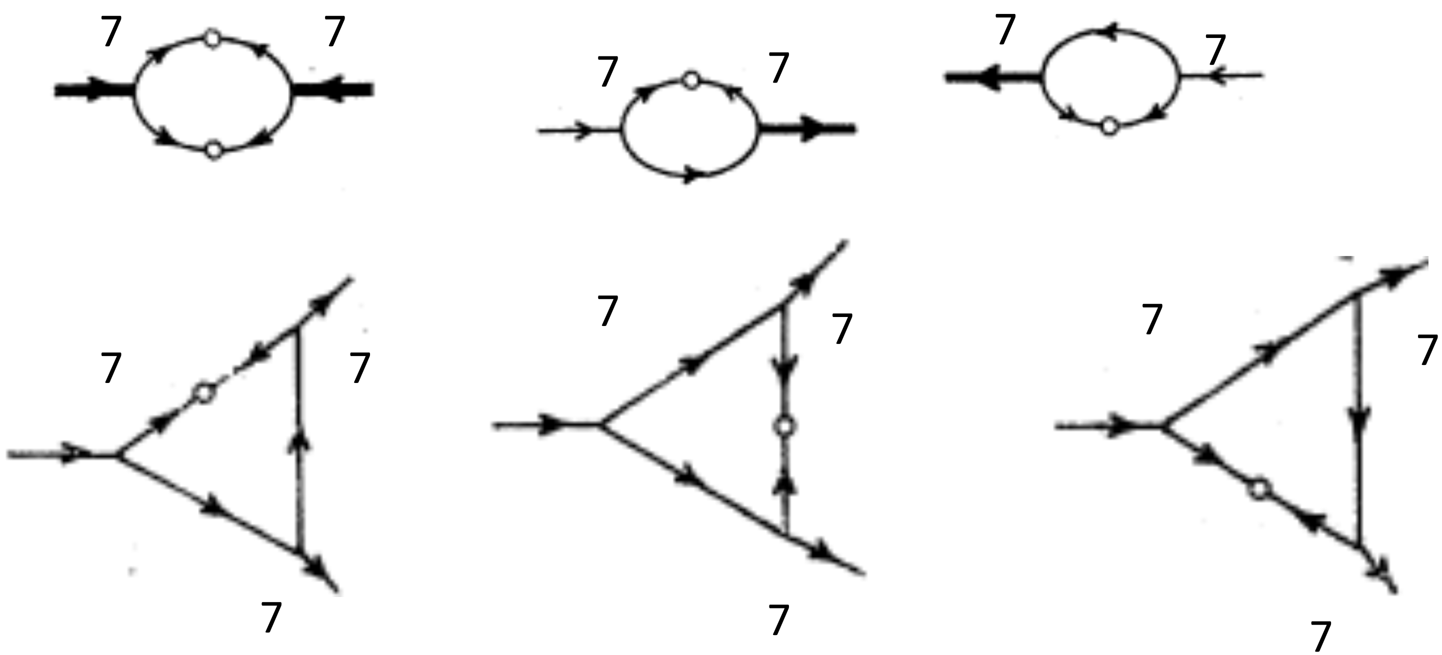
Expanded equations of motion:

7 (count 'em, 7) non-linearities

$$\partial_t \vec{v}_\perp + \gamma \partial_\parallel \vec{v}_\perp + \lambda_1^0 (\vec{v}_\perp \cdot \vec{\nabla}_\perp) \vec{v}_\perp$$
$$= -g_1 \delta\rho \partial_\parallel \vec{v}_\perp - g_2 \vec{v}_\perp \partial_\parallel \delta\rho - \frac{c_0^2}{\rho_0} \vec{\nabla}_\perp \delta\rho - g_3 \vec{\nabla}_\perp (\delta\rho^2)$$
$$+ D_B \vec{\nabla}_\perp (\vec{\nabla}_\perp \cdot \vec{v}_\perp) + D_T \nabla_\perp^2 \vec{v}_\perp + D_\parallel \partial_\parallel^2 \vec{v}_\perp$$
$$+ \nu_t \partial_t \vec{\nabla}_\perp \delta\rho + \nu_\parallel \partial_\parallel \vec{\nabla}_\perp \delta\rho + \vec{f}_\perp \quad (2.18)$$

$$\partial_t \delta\rho + \rho_o \vec{\nabla}_\perp \cdot \vec{v}_\perp + w_1 \vec{\nabla}_\perp \cdot (\vec{v}_\perp \delta\rho) + v_2 \partial_\parallel \delta\rho$$
$$= D_{\rho\parallel} \partial_\parallel^2 \delta\rho + D_{\rho\perp} \nabla_\perp^2 \delta\rho + D_{\rho v} \partial_\parallel (\vec{\nabla}_\perp \cdot \vec{v}_\perp)$$
$$+ \phi \partial_t \partial_\parallel \delta\rho + w_2 \partial_\parallel (\delta\rho^2) + w_3 \partial_\parallel (|\vec{v}_\perp|^2),$$

Feynmann graphs:



Number of graphs=(7^2)x3+(7^3)x3=1176!

Long-ranged order in d=2

- Stabilized by this breakdown of hydrodynamics (damps out noise induced fluctuations (negative feedback)
- z(2)<2 => faster damping of fluctuations
- =>LRO
- We know this happens because we know
- z(2)<2, even though we don't know it's actual value

But how to determine scaling laws?

Consider Incompressible flock:
six of seven NL's involve ρ

=>All gone if we fix ρ

Can actually get exact scaling exponents
in all d in this case (with very little work!)

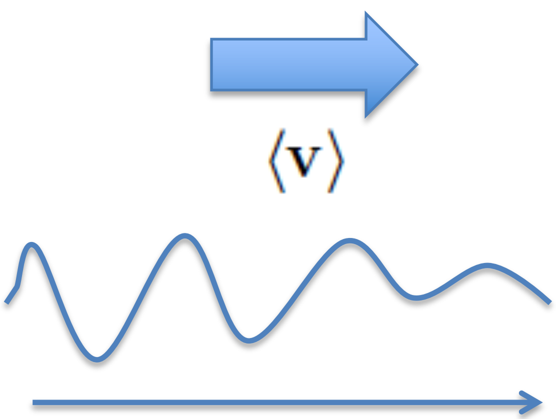
$$z = \frac{2(d+1)}{5}\ ,\ \zeta = \frac{d+1}{5}\ ,\ \chi = \frac{3-2d}{5}$$

Definitions of other exponents:

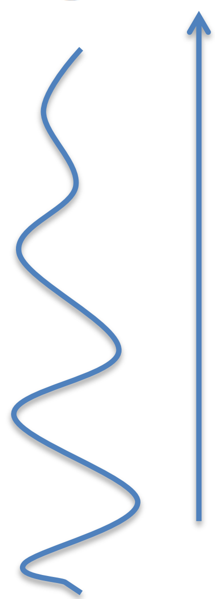
Anisotropy exponent

Roughness exponent

$\zeta\ \chi$



$\langle \mathbf{v} \rangle$



$L_a \propto \lambda^{z(d)}$

$$L_a \propto \lambda^{z(d)/\zeta(d)} = \lambda^2$$

$$G_D(g_2) \equiv \frac{(d-2)}{2(d-1)}\frac{1}{g_2^2}\left[1+\frac{1}{\sqrt{g_2+1}}-\frac{2\sqrt{2}}{\sqrt{g_2+2}}\right]$$

$$= \frac{1}{g_2^2}\left[\frac{1}{3}+\frac{1}{3\sqrt{g_2+1}}-\frac{2\sqrt{2}}{3\sqrt{g_2+2}}\right] , \qquad (d=4)$$

$$G_{\mu_1}(g_2) \equiv \frac{2}{d^2-1}\left(\frac{2d^2-6d+3}{32}+\frac{(d+3)\sqrt{2}}{g_2^2(g_2+2)^{3/2}}-\frac{1}{g_2^2}-\frac{d+1}{2g_2^2\sqrt{g_2+1}}+\frac{d-3}{2g_2}+\frac{d+15}{2\sqrt{2}g_2(g_2+2)^{3/2}}\right.$$

$$\left.+\frac{3}{\sqrt{2}(g_2+2)^{3/2}}-\frac{d+1}{4g_2\sqrt{g_2+1}}+\frac{3-d}{2\sqrt{2}g_2\sqrt{g_2+2}}\right)$$

$$G_{\mu_2}(g_2) \equiv \frac{2}{(d^2-1)g_2}\left(-\frac{(3d-1)\sqrt{2}}{g_2^2(g_2+2)^{3/2}}+\frac{(d-1)}{g_2^2}+\frac{(d+1)}{2g_2^2\sqrt{g_2+1}}+\frac{(d^2-4d+3)\sqrt{2}}{4g_2\sqrt{g_2+2}}-\frac{(d^2-7d+8)}{4g_2}\right.$$

$$\left.+\frac{d+1}{64(g_2+1)^{3/2}}+\frac{(13-15d)}{2\sqrt{2}g_2(g_2+2)^{3/2}}-\frac{3(d-1)}{\sqrt{2}(g_2+2)^{3/2}}+\frac{(d+1)}{4g_2\sqrt{g_2+1}}+\frac{2d^2-9d+11}{32}\right)$$