Effective field theories for

Fluid Dynamics ~~from Black Holes~~ and Enstrophy

Natalia Pinzani Fokeeva - KU Leuven

Leiden - October 2019

~~with J. de Boer and M. P. Heller: 1812.06093 + w.i.p. also with Ricardo Espindola~~

with K. Jensen, R. Marjieh, and A. Yarom 1804.04654 + w.i.p. with R. Marjieh, and A. Yarom

Effective field theories for

Fluid Dynamics

Natalia Pinzani Fokeeva - KU Leuven

Leiden - October 2019

with K. Jensen, R. Marjieh, and A. Yarom 1804.04654

The conventional formalism

- Equations of motion $\nabla_\mu T^{\mu\nu} = 0$ $\nabla_\mu J^\mu = 0$
- Constitutive relations
$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}(u^\mu, T, \partial) \\ &= P(T)g_{\mu\nu} + (P'(T)T)u_\mu u_\nu + \eta\partial_\mu u_\nu + \dots \end{aligned}$$
- 2nd Law of Thermodynamics $\nabla_\mu S^\mu \geq 0$
- Onsager relations
- Statistical fluctuations
- ... ??

The conventional formalism

- Equations of motion $\nabla_\mu T^{\mu\nu} = 0$ $\nabla_\mu J^\mu = 0$
- Constitutive relations
$$T^{\mu\nu} = T^{\mu\nu}(u^\mu, T, \partial) = P(T)g_{\mu\nu} + (P'(T)T)u_\mu u_\nu + \eta\partial_\mu u_\nu + \ldots$$
- 2nd Law of Thermodynamics $\nabla_\mu S^\mu \geq 0$
- Onsager relations
- Statistical fluctuations
- ... ??

How viscous?

The conventional formalism

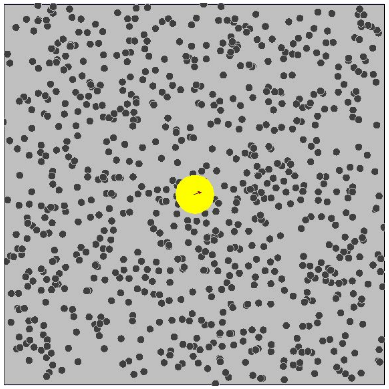
- Equations of motion $\nabla_\mu T^{\mu\nu} = 0$ $\nabla_\mu J^\mu = 0$
- Constitutive relations
$$T^{\mu\nu} = T^{\mu\nu}(u^\mu, T, \partial) = P(T) g_{\mu\nu} + (P'(T)T) u_\mu u_\nu + \eta \partial_\mu u_\nu + \dots$$
- 2nd Law of Thermodynamics $\nabla_\mu S^\mu \geq 0$

  → $\eta \geq 0$ + Some transport coefficients are vanishing at second order
- Onsager relations
- Statistical fluctuations
- ... ??

The conventional formalism

- Equations of motion $\nabla_\mu T^{\mu\nu} = 0$ $\nabla_\mu J^\mu = 0$
- Constitutive relations
$$T^{\mu\nu} = T^{\mu\nu}(u^\mu, T, \partial) = P(T)g_{\mu\nu} + (P'(T)T)u_\mu u_\nu + \eta\partial_\mu u_\nu + \ldots$$
- 2nd Law of Thermodynamics $\nabla_\mu S^\mu \geq 0$
- Onsager relations
- Statistical fluctuations
- ... ??

E.g. Brownian motion



Is this set of constraints sufficient?

Is this set of constraints sufficient?

No.

Using Schwinger-Keldysh effective field theory techniques we are able to determine that additional constraints exist

Schwinger-Keldysh EFTs

- The Schwinger-Keldysh path integral

UV:

$$Z[A_1, A_2] = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 e^{iS[\phi_1, A_1] - iS[\phi_2, A_2]}$$

$$= Tr(\mathcal{U}[A_1]\rho(-\infty)\mathcal{U}^\dagger[A_2])$$

↓

IR:

$$Z[A_1, A_2] = \int \mathcal{D}\xi_1 \mathcal{D}\xi_2 e^{iS_{eff}[\xi_1, \xi_2; A_1, A_2; \rho]}$$

Schwinger-Keldysh EFTs

- The Schwinger-Keldysh path integral

UV:

$$Z[A_1, A_2] = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 e^{iS[\phi_1, A_1] - iS[\phi_2, A_2]}$$

$$= Tr(\mathcal{U}[A_1]\rho(-\infty)\mathcal{U}^\dagger[A_2])$$

IR:

$$Z[A_1, A_2] = \int \mathcal{D}\xi_1 \mathcal{D}\xi_2 e^{iS_{eff}[\xi_1, \xi_2; A_1, A_2; \rho]}$$

1. Symmetries
2. Degrees of freedom

The symmetries

The symmetries can be inferred from the microscopic definition

$$Z[A_1, A_2] = Tr(\mathcal{U}[A_1]\rho(-\infty)\mathcal{U}^\dagger[A_2])$$

2) SK symmetry: $Z[A_1 = A_2 = A_r] = 1$

3) Reality condition: $Z[A_1, A_2]^* = Z[A_2, A_1]$

4) KMS symmetry: (for thermal states $\rho = e^{-\beta H}$ + CPT)

$$Z[A_1(t_1), A_2(t_2)] = Z[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$$

5) Constraint $|Z|^2 \leq 1$

[M. Crossley, P. Glorioso, H. Liu (1511.03646)]

Schwinger-Keldysh positivity

- The constraint $|Z|^2 \leq 1$ → $\mathrm{Im} S_{eff} \geq 0$

A.	$\mathrm{Im} S_{eff} = 0$	$\nabla_\mu S^\mu = 0$	Non dissipative
B.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu \geq 0$	Dissipative
C.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu = 0$	Pseudo-dissipative
D.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu = 0$	Exceptional

Example

- The constraint $|Z|^2 \leq 1$ → $\mathrm{Im} S_{eff} \geq 0$

A.	$\mathrm{Im} S_{eff} = 0$	$\nabla_\mu S^\mu = 0$	Non dissipative
B.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu \geq 0$	Dissipative
C.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu = 0$	Pseudo-dissipative
D.	$\mathrm{Im} S_{eff} \geq 0$	$\nabla_\mu S^\mu = 0$	Exceptional

Example

- Example of exceptional transport

$$T_{ij} = P(T)P_{ij} + \epsilon(T)u_i u_j + \left(\gamma(T,\mu) - \gamma(T,-\mu)\right)\left(P_{ij}\sigma^2 - 2\Theta\sigma_{ij}\right)$$

Example

- Example of exceptional transport

$$T_{ij} = P(T)P_{ij} + \epsilon(T)u_i u_j + (\gamma(T,\mu) - \gamma(T,-\mu))\left(P_{ij}\sigma^2 - 2\Theta\sigma_{ij}\right)$$

Projector

$$P_{ij} = g_{ij} + u_i u_j$$

Shear

$$\sigma_{kl} = P^i_k P^j_l (\nabla_i u_j + \nabla_j u_i) - \frac{2}{d} P_{ij}\Theta$$

Expansion

$$\Theta = \nabla_i u^i$$

Example

- Example of exceptional transport

$$T_{ij} = P(T)P_{ij} + \epsilon(T)u_iu_j + (\gamma(T,\mu) - \gamma(T,-\mu))\left(P_{ij}\sigma^2 - 2\Theta\sigma_{ij}\right)$$

$$P_{ij} = g_{ij} + u_iu_j$$

Projector

Shear

Expansion

$$\Theta = \nabla_i u^i$$

$$\sigma_{kl} = P^i_k P^j_l(\nabla_i u_j + \nabla_j u_i) - \frac{2}{d}P_{ij}\Theta$$

- $\mathrm{Im}S_{eff} \geq 0 \qquad \nabla_\mu S^\mu = 0$

$$\rightarrow \int d^{d+1}\sigma\sqrt{-g}\gamma^-\left(P^{ij}P^{kl} + 2\Theta\left(\frac{1}{d}P^{ij}P^{kl} - P^{i(k}P^{l)j}\right)\right)g_{a\,ij}g_{a\,kl} \geq 0$$

$$\rightarrow \gamma^- = \gamma(T,\mu) - \gamma(T,-\mu) = 0$$

Is this set of constraints sufficient?

No.

Using Schwinger-Keldysh effective field theory techniques we are able to determine that additional constraints exist:
Schwinger-Keldysh positivity

Outlook

- A more systematic analysis?
- What would be the holographic counterpart?

Enstrophy

Natalia Pinzani Fokeeva - KU Leuven

Leiden - October 2019

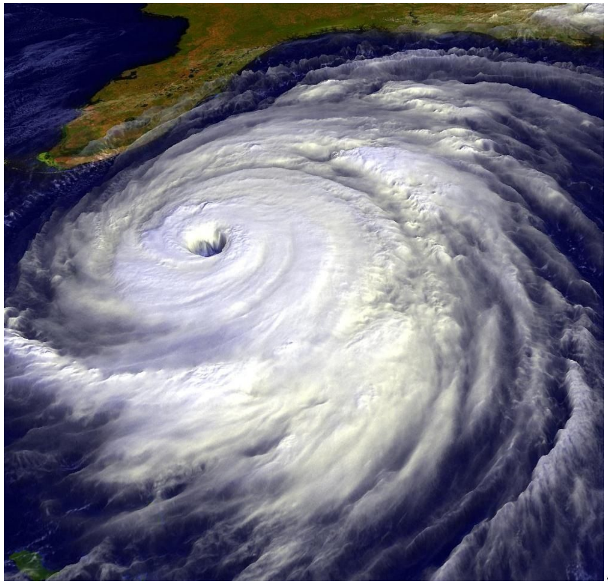
w.i.p. with R. Marjieh, and A. Yarom

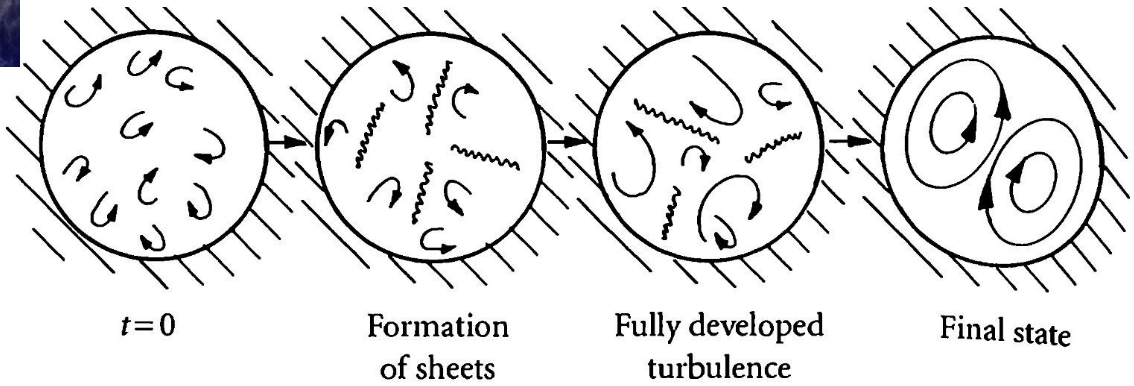
Motivation

- 3+1 turbulence → **direct** energy cascade (from big to small scales)

Motivation

- 2+1 turbulence ➝ **inverse** energy cascade (from small to large scales)





Motivation

- 2+1 turbulence → **inverse** energy cascade
- In non relativistic fluid flows, its origin can be traced back to the existence of an approximately conserved **enstrophy charge:**

$$\Omega = \int \omega^{ij}\omega_{ij} d^d x$$

$$\partial_t \Omega = \int \omega^{ij}\sigma_j^k \omega_{ki}\, d^d x - \frac{1}{R} P$$

Motivation

- 2+1 turbulence → **inverse** energy cascade
- In non relativistic fluid flows, its origin can be traced back to the existence of an approximately conserved **enstrophy charge:**

$$\Omega = \int \omega^{ij}\omega_{ij} d^d x$$

vorticity: $\omega_{ij} = \partial_i v_j - \partial_j v_i$

shear: $\sigma_{ij} = \partial_i v_j + \partial_j v_i$

$$\partial_t \Omega = \int \omega^{ij} \sigma^k_j \omega_{ki} \, d^d x - \frac{1}{R} P$$

Vortex-stretching term

Motivation

- 2+1 turbulence → **inverse** energy cascade
- In non relativistic fluid flows, its origin can be traced back to the existence of an approximately conserved **enstrophy charge**

$$\Omega = \int \omega^{ij}\omega_{ij} d^d x$$

$$\partial_t \Omega = \int \omega^{ij}\sigma_j^k \omega_{ki}\, d^d x - \frac{1}{R} P$$

In d=2 spatial dimensions the Vortex-stretching term is vanishing

Motivation

- 2+1 turbulence → **inverse** energy cascade
- In non relativistic fluid flows, its origin can be traced back to the existence of an approximately conserved **enstrophy charge**

$$\Omega = \int \omega^{ij}\omega_{ij} d^d x$$

- In d=2:

Palinstrophy: $$P = \int \partial_k \omega^{ij}\, \partial^k \omega_{ij}\, d^d x$$

$$\partial_t \Omega = -\frac{1}{R} P$$

Reynolds number

[Kraichnan 1967, Leith 1968, Batchelor 1969]

Motivation

- 2+1 turbulence → **inverse** energy cascade
- In **non relativistic** fluid flows, its origin can be traced back to the existence of an approximately conserved **enstrophy charge**
- **Relativistic** generalization to uncharged, conformal fluid flows

[F. Carrasco, L. Lehner , Robert C. Myers, O. Reula, A. Singh 1210.6702]

  - Numerical simulations in holography
  - Astrophysics?
  - Heavy-ion collisions?

1. An enstrophy current for charged, relativistic fluid flows?

2. Can it be derived from a symmetry principle?

1. An enstrophy current for charged, relativistic fluid flows?

   Uncharged: yes
   Charged: only for restricted equations of states

2. Can it be derived from a symmetry principle?

   Yes, from an effective action for ideal fluid dynamics

1. An enstrophy current for charged, relativistic fluid flows?

   Uncharged: yes
   Charged: only for restricted equations of states

2. Can it be derived from a symmetry principle?

   Yes, from an effective action for ideal fluid dynamics

Enstrophy current for **uncharged** fluids

- Consider a closed 2-form

$$\Omega_{\mu\nu} = \nabla_\mu (T u_\nu) - \nabla_\nu (T u_\mu)$$

- On-shell it satisfies

$$\Omega_{\mu\nu} u^\nu = 0$$

- The enstrophy current

$$J^\mu = \frac{1}{s} \Omega_{\alpha\beta} \Omega^{\alpha\beta} u^\mu$$

- Conservation equation

$$\nabla_\mu J^\mu = -\frac{1}{s^2} \nabla_\mu (s u^\mu) \Omega^2 + \frac{2}{s} \Omega^{\alpha\beta} \nabla_\mu (u^\mu \Omega_{\alpha\beta})$$

Enstrophy current for **uncharged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- Conservation equation

$$\nabla_\mu J^\mu = -\frac{1}{s^2}\nabla_\mu(su^\mu)\Omega^2 + \frac{2}{s}\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta})$$

Enstrophy current for **uncharged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- Conservation equation

$$\nabla_\mu J^\mu = -\frac{1}{s^2}\nabla_\mu(su^\mu)\Omega^2 + \frac{2}{s}\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta})$$

Vanishing on-shell

Enstrophy current for **uncharged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\, u^\mu$$

- Conservation equation

$$\nabla_\mu J^\mu = -\frac{1}{s^2}\nabla_\mu(su^\mu)\Omega^2 + \frac{2}{s}\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta})$$

Vanishing on-shell

- It is straightforward to show

$$\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta}) = \frac{1}{2}\Omega^{\alpha\beta}\left(\sigma^\mu_\alpha\Omega_{\mu\beta} + \sigma^\mu_\beta\Omega_{\alpha\mu}\right) + \left(1 - \frac{2}{d}\right)\Theta\Omega^2$$

Relativistic generalization of the vortex-stretching term

Enstrophy current for **uncharged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\, u^\mu$$

- Conservation equation

$$\nabla_\mu J^\mu = -\frac{1}{s^2}\nabla_\mu(su^\mu)\Omega^2 + \frac{2}{s}\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta})$$

Vanishing on-shell

- It is straightforward to show

$$\Omega^{\alpha\beta}\nabla_\mu(u^\mu\Omega_{\alpha\beta}) = \frac{1}{2}\Omega^{\alpha\beta}\left(\sigma^\mu_\alpha\Omega_{\mu\beta} + \sigma^\mu_\beta\Omega_{\alpha\mu}\right) + \left(1-\frac{2}{d}\right)\Theta\Omega^2$$

In 2 spatial dimensions these terms are vanishing on-shell

Mini summary

- The enstrophy current

$$J^\mu = \frac{1}{s}\Omega_{\alpha\beta}\Omega^{\alpha\beta}u^\mu$$

$$\Omega_{\mu\nu} = \nabla_\mu(Tu_\nu) - \nabla_\nu(Tu_\mu)$$

$$\Omega_{\mu\nu}u^\nu = 0$$ on-shell

- Conserved on-shell in 2 spatial dimensions

$$\nabla_\mu J^\mu = \mathcal{O}(\partial^3)$$

1. An enstrophy current for charged, relativistic fluid flows?

Uncharged: yes
Charged: only for restricted equations of states

2. Can it be derived from a symmetry principle?

yes, from an effective action for ideal fluid dynamics

Enstrophy current for **charged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- Another 2-form

$$\Omega_{\alpha\beta} = \nabla_\alpha(Tf(T,\mu/T)u_\beta) - \nabla_\beta(Tf(T,\mu/T)u_\alpha) + c\,F_{\alpha\beta}$$

- It satisfies on-shell

$$\Omega_{\alpha\beta}u^\beta = \left(f\,\frac{\rho T}{P+\epsilon} - \frac{\partial f}{\partial(\mu/T)}\right) T D^\perp_\alpha(\mu/T) - T\frac{\partial f}{\partial T} D^\perp_\alpha T - \left(f\,\frac{\rho T}{P+\epsilon} - c\right) F_{\alpha\beta}u^\beta$$

$$D^\perp_\alpha = (\delta^\mu_\alpha + u^\mu u_\alpha)\partial_\mu$$

Enstrophy current for **charged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- It satisfies on-shell

$$\Omega_{\alpha\beta}u^\beta = \left(f\,\frac{\rho T}{P+\epsilon} - \frac{\partial f}{\partial(\mu/T)}\right) T D^\perp_\alpha(\mu/T) - T\frac{\partial f}{\partial T} D^\perp_\alpha T - \left(f\,\frac{\rho T}{P+\epsilon} - c\right) F_{\alpha\beta}u^\beta$$

Enstrophy current for **charged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- It satisfies on-shell

$$\Omega_{\alpha\beta}u^\beta = \left(f\,\frac{\rho T}{P+\epsilon} - \frac{\partial f}{\partial(\mu/T)}\right) T D^\perp_\alpha(\mu/T) - T\frac{\partial f}{\partial T} D^\perp_\alpha T - \left(f\,\frac{\rho T}{P+\epsilon} - c\right) F_{\alpha\beta}u^\beta$$

$$\frac{\partial f}{\partial T} = 0$$

Enstrophy current for **charged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- It satisfies on-shell

$$\Omega_{\alpha\beta}u^\beta = \left(f\,\frac{\rho T}{P+\epsilon} - \frac{\partial f}{\partial(\mu/T)}\right)T D^\perp_\alpha(\mu/T) - T\frac{\partial f}{\partial T}D^\perp_\alpha T - \left(f\,\frac{\rho T}{P+\epsilon} - c\right)F_{\alpha\beta}u^\beta$$

$$\frac{\partial f}{\partial T} = 0$$

In the absence of external sources: $P(T,\mu) = p(Tf(\mu/T))$

Enstrophy current for **charged** fluids

- The enstrophy current

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

- It satisfies on-shell (which is in general non-vanishing!)

$$\Omega_{\alpha\beta}u^\beta = \left(f\,\frac{\rho T}{P+\epsilon} - \frac{\partial f}{\partial(\mu/T)}\right) T D^\perp_\alpha(\mu/T) - T\frac{\partial f}{\partial T} D^\perp_\alpha T - \left(f\,\frac{\rho T}{P+\epsilon} - c\right) F_{\alpha\beta}u^\beta$$

$$\frac{\partial f}{\partial T} = 0$$

In the absence of external sources: $P(T,\mu) = p(T f(\mu/T))$

Otherwise: $f = c_0 + c\,\mu/T$

Mini summary

- The enstrophy current for charged fluids

$$J^{\mu} = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^{\mu}$$

$$\Omega_{\alpha\beta} = \nabla_{\alpha}(Tf(\mu/T)u_{\beta}) - \nabla_{\beta}(Tf(\mu/T)u_{\alpha}) + cF_{\alpha\beta}$$

In the absence of external sources: $P(T,\mu) = p(Tf(\mu/T))$

Otherwise: $f = c_0 + c\,\mu/T$

- Conservation equation in 2 spatial dimensions

$$\nabla_{\mu}J^{\mu} = \mathcal{O}(\partial^4)$$

Mini summary

- The enstrophy current for charged fluids

$$J^\mu = \frac{1}{s}\,\Omega_{\alpha\beta}\Omega^{\alpha\beta}\,u^\mu$$

$$\Omega_{\alpha\beta} = \nabla_\alpha(Tf(\mu/T)u_\beta) - \nabla_\beta(Tf(\mu/T)u_\alpha) + cF_{\alpha\beta}$$

In the absence of external sources: $P(T,\mu) = p(Tf(\mu/T))$

Otherwise: $f = c_0 + c\,\mu/T$

- Conservation equation in 2 spatial dimensions

$$\nabla_\mu J^\mu = \mathcal{O}(\partial^4)$$

- A family of conserved currents $J^\mu_n = \dfrac{1}{(as+b\rho)^{2n-1}}\,\Omega^n\,u^\mu$

1. An enstrophy current for charged, relativistic fluid flows?

   Uncharged: yes
   Charged: only for restricted equations of states

2. Can it be derived from a symmetry principle?

   yes, from an effective action for ideal fluid dynamics

The ideal fluid effective action

- The leading order effective action for fluid dynamics

$$S_{eff} = \int \sqrt{-g}\, P(T,\mu)\, d^{d+1}\sigma$$

- The invariants

$$T = \frac{1}{\sqrt{-\beta^i \beta^j g_{ij}}} \qquad \mu/T = \beta^i B_i + \Lambda_\beta$$

- The pullback sources

$$g_{ij}(\sigma) = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X(\sigma))$$
$$B_i(\sigma) = \partial_i X^\mu B_\mu(X(\sigma)) + \partial_i C(\sigma)$$

The ideal fluid effective action

- The leading order effective action for fluid dynamics

$$S_{eff} = \int \sqrt{-g}\, P(T,\mu)\, d^{d+1}\sigma$$

- The invariants

$$T = \frac{1}{\sqrt{-\beta^i \beta^j g_{ij}}} \qquad \mu/T = \beta^i B_i + \Lambda_\beta$$

- The pullback sources

External sources

$$g_{ij}(\sigma) = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X(\sigma))$$

$$B_i(\sigma) = \partial_i X^\mu B_\mu(X(\sigma)) + \partial_i C(\sigma)$$

The ideal fluid effective action

- The leading order effective action for fluid dynamics

$$S_{eff} = \int \sqrt{-g}\, P(T, \mu)\, d^{d+1}\sigma$$

Initial state data

- The invariants

$$T = \frac{1}{\sqrt{-\beta^i \beta^j g_{ij}}} \qquad \mu/T = \beta^i B_i + \Lambda_\beta$$

External sources

- The pullback sources

$$g_{ij}(\sigma) = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X(\sigma))$$

$$B_i(\sigma) = \partial_i X^\mu B_\mu(X(\sigma)) + \partial_i C(\sigma)$$

The ideal fluid effective action

- The leading order effective action for fluid dynamics

$$S_{eff} = \int \sqrt{-g}\, P(T,\mu)\, d^{d+1}\sigma$$

Initial state data

- The invariants

$$T = \frac{1}{\sqrt{-\beta^i \beta^j g_{ij}}} \qquad \mu/T = \beta^i B_i + \Lambda_\beta$$

- The pullback sources

External sources

$$g_{ij}(\sigma) = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X(\sigma))$$

$$B_i(\sigma) = \partial_i X^\mu B_\mu(X(\sigma)) + \partial_i C(\sigma)$$

Dynamical fields

Enstrophy from symmetry

- The leading order effective action for fluid dynamics

$$S_{eff} = \int \sqrt{-g}\, P(T,\mu)\, d^{d+1}\sigma$$

- The transformation of the dynamical fields

$$\delta X^\mu = \frac{\Omega^2}{Ts^2}u^\mu - \frac{2}{sp'}P^{\mu\alpha}\left(2\nabla_\nu \Omega^\nu{}_\alpha + \frac{\Theta E_\alpha}{p'} + 2\Omega_{\nu\alpha}a^\nu + \frac{2}{s}\left(\frac{\partial s}{\partial T}\nabla_\nu T + \frac{\partial s}{\partial \mu}\nabla_\nu \mu\right)\Omega_\alpha{}^\nu\right)$$

$$\delta C = -\frac{\mu\Omega^2}{s^2 T}\,,$$

- Leads to the conserved Noether current

$$\tilde{J}^\mu = J^\mu - \frac{4}{sp'}\Omega^{\mu\nu}E_\nu$$

Equations of motion

1. An enstrophy current for charged, relativistic fluid flows?

   Uncharged: yes
   Charged: only for restricted equations of states

2. Can it be derived from a symmetry principle?

   yes, from an effective action for ideal fluid dynamics

Outlook:

- $\nabla_\mu J^\mu \leq 0$?
- Does that imply inverse energy cascade for relativistic fluids?
- The symmetry responsible for enstrophy conservation in AdS4 can be related to a near horizon supertranslation [w.i.p.]
- What is the corresponding geometric quantity that grows/decreases?

[see perhaps A. Adams, P. M. Chesler, and H. Liu 1307.7267]

Thank you!

Natalia Pinzani Fokeeva - KU Leuven

Leiden - October 2019

with K. Jensen, R. Marjieh, and A. Yarom 1804.04654 + w.i.p. with R. Marjieh, and A. Yarom