Superfluids as anomalies

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Motivations

- Higher-form symmetries give a symmetry based way of thinking about magnetohydrodynamics [Grozdanov, Hofman, Iqbal, '17], elasticity theory [Grozdanov, Poovuttikul, '18; Armas, Jain, '19], …

- Extend Landau paradigm for higher-form symmetries. Phase transition that were though to be beyond Landau are actually within Landau but for higher-form symmetries [Gaiotto, Kapustin, Seiberg, Willet, '14]. How does the BKT transition fit in this picture ?

Landau paradigm and BKT phase transition

Landau Theory

Landau paradigm : The free energy can be written as a Taylor expansion of the order parameter $o(T)$.

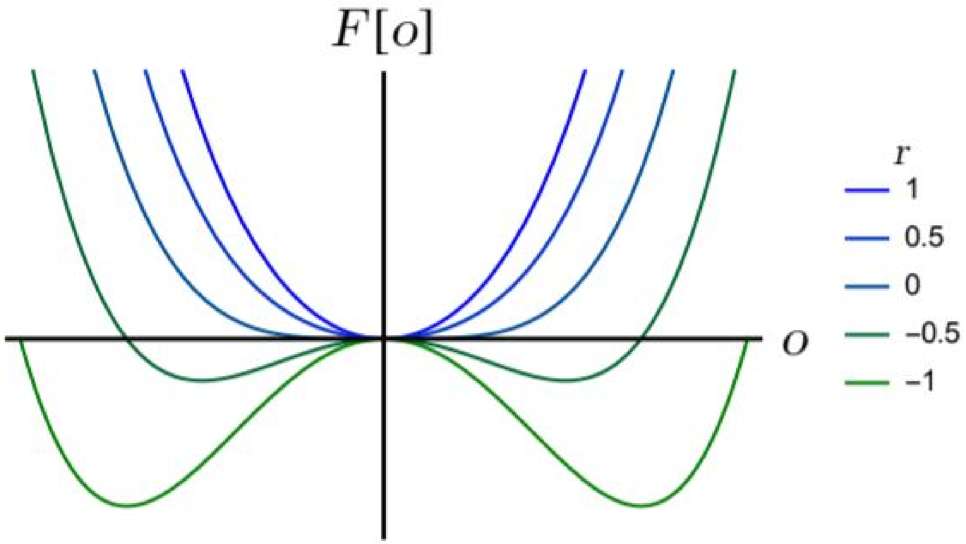
$$\mathcal{F}_L(o,T) = \mathcal{F}(0,T) + \frac{1}{2}r(T)o^2 + \frac{1}{4}u(T)o^4 + \ldots$$

The free energy functional $\mathcal{F}_L(o,T)$ should be bounded from below, which is guaranteed if the highest order term is positive.

- If r(T) is positive, the minimum of the free energy is at $o(T) = 0$
- If r(T) is negative, the minimum is at

$$o(T) = \pm\sqrt{\frac{|r(T)|}{u(T)}}$$

⇒Phase transition : r(T) changes sign.



Our proposal

Phase transitions are described by an order parameter, which is finite on the ordered phase side of the transition (spontaneously broken phase) and zero on the disordered phase. At the same time, we want to label the symmetries that are spontaneously broken.

We will see

Spontaneous Symmetry Breaking $\Rightarrow$ $\mathrm{Emergence\ of\ extra\ symmetries}$

Our proposal : Label phases by their global symmetries only, without explicit specification of how they are realized (linearly or non-linearly).

Example: BKT phase transition

Consider the following EFT for a U(1) compact boson of radius $R \sim \frac{1}{g}$.

$$S = \frac{1}{g^2} \int d^2x (\partial\phi)^2$$

This has a shift symmetry $\phi \to \phi + c$, with a conserved current

$$J = d\phi, \qquad d \star J = 0$$

There is also a dual current, which is topological (always conserved)

$$\tilde{J} = d\tilde{\phi} = \star d\phi, \qquad d \star \tilde{J} \propto d^2\phi = 0$$

This is really $U(1) \times \widetilde{U(1)}$.

There are no SSB in $d = 2$. How do we explain this gapless theory ?

BKT phase transition

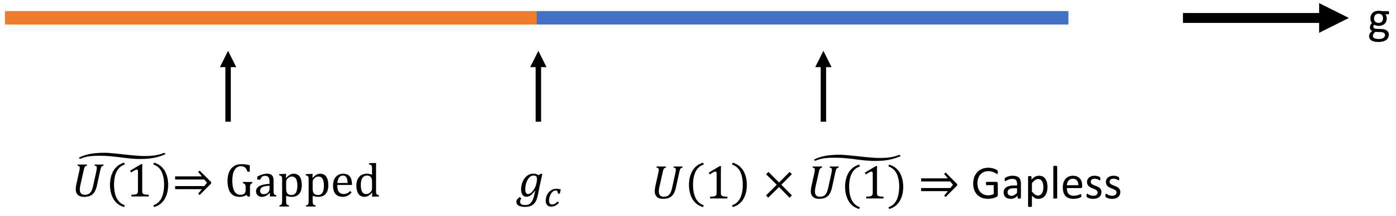
Imagine that J is accidental in the IR, and write an interaction that preserves the topological symmetry as

$$S = \int d^2x \frac{1}{g^2}(\partial\phi)^2 + \lambda\cos(\phi) + \ldots$$

λ need some mass dimension, which makes the theory gapped.

$$[\lambda] = 2 - [\cos(\phi)] = 2 - g^2$$

For $g_c = \sqrt{2}$, λ is marginal while for $g > g_c$, it is irrelevant and the IR become gapless again.



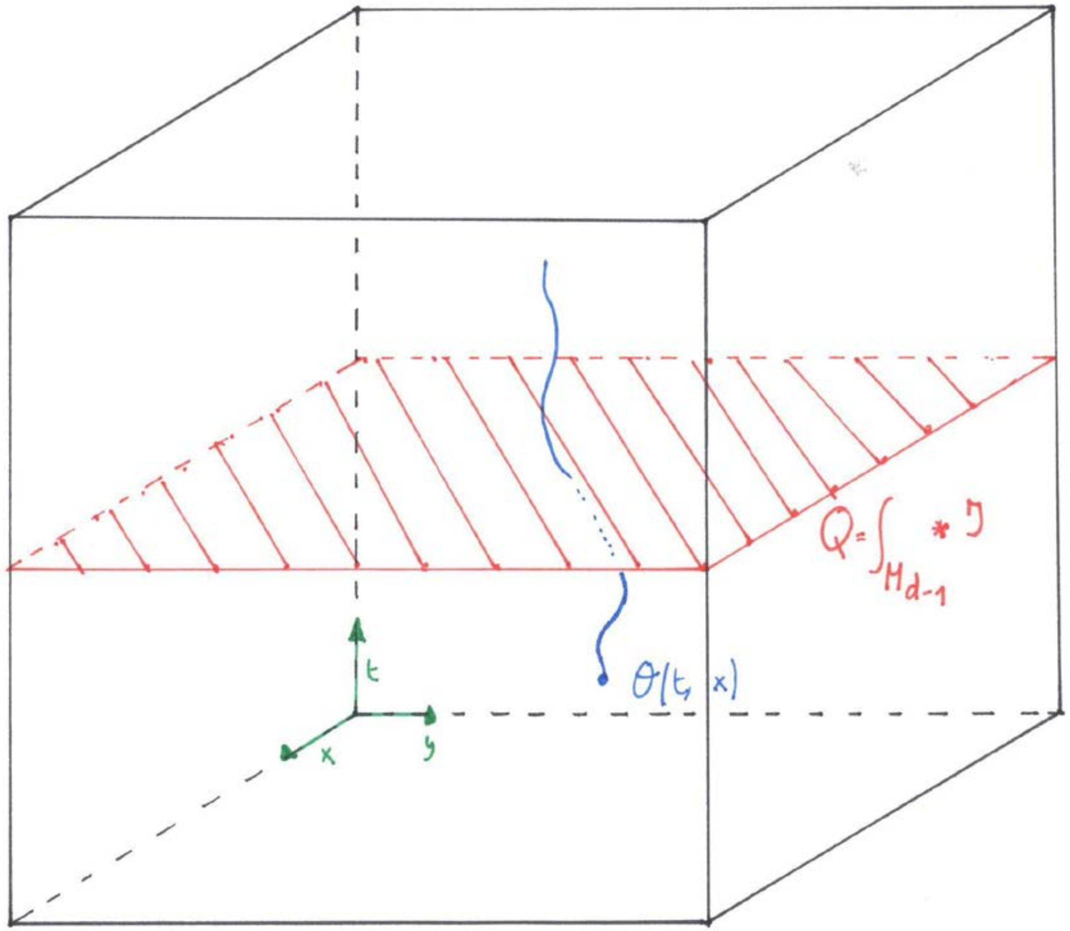
The symmetry structure is different. This happens without SSB. The phase is protected by the fact that U(1) symmetry is in fact **anomalous**.

Plan

1. Introduction to higher-form symmetries
2. Superfluids
3. Goldstone theorem
4. Applications : Hydrodynamics

Higher-form symmetries

0-form symmetry : Regular symmetry



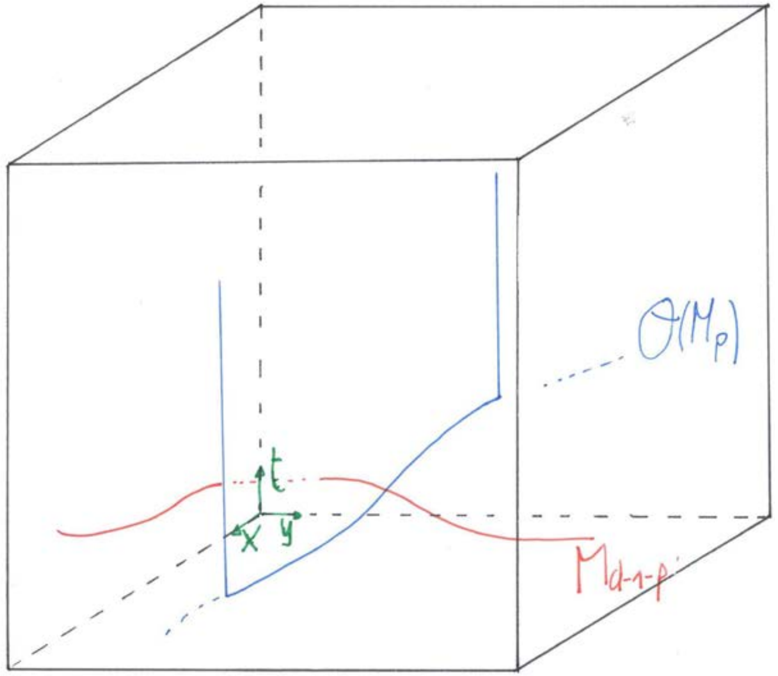
- Point-like charged operators $\vartheta(x, t)$ creating charged excitations.
- Charge operators

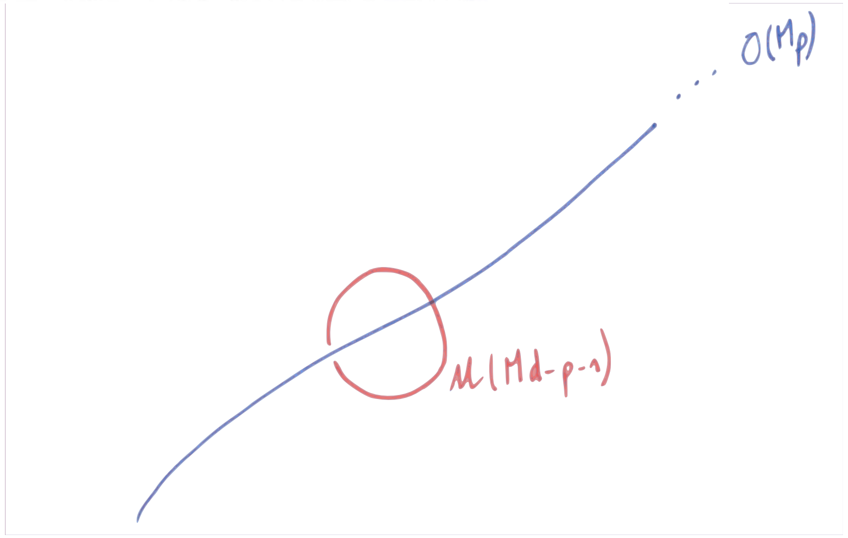
$$Q(M_{d-1}) = \int_{M_{d-1}} \star J$$

- Charge operators can be non-Abelian
- For connected Lie group, symmetry operators are

$$U_\alpha = e^{i\alpha^a Q_a}$$

Higher (p)-form symmetry [Gaiotto, Kapustin, Seiberg, Willet, ‘14]





- Extended charged operators $\vartheta(M_p)$ creating conserved excitations.
- Noether current is a (p+1)-form
- Charge operators that counts excitations are

$$Q(M_{d-1-p}) = \int_{M_{d-1-p}} \star J$$

- Charge operators are Abelian
- Symmetry operators are

$$U_{g=e^{i\alpha}} = e^{i\alpha Q(M_{d-1-p})}$$

- Measure the charge of the operator $\vartheta(M_p)$ by wrapping it

$$U_g(M_{d-1-p})\mathcal{O}(M_p) = g^q \mathcal{O}(M_p)$$

Superfluids

Superfluids EFT

Consider the following effective action for a massless phase

$$S = -\int d^dx \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}(\partial\phi)^4 + \ldots$$

It has a regular U(1) shift symmetry $\phi \to \phi + c$ with current

$$J_\mu = \partial_\mu\phi(1 + \lambda(\partial\phi)^2 + \ldots)$$

We also have a topological (identically conserved) higher form current

$$\partial_{[\mu}\partial_{\nu]}\phi = 0 = \partial_{\mu_1}(\underbrace{\epsilon^{\mu_1\cdots\mu_{d-1}\lambda}\partial_\lambda\phi})$$

$K^{\mu_1\cdots\mu_{d-1}}$

So, we see that we have a (d-2)-form symmetry with current

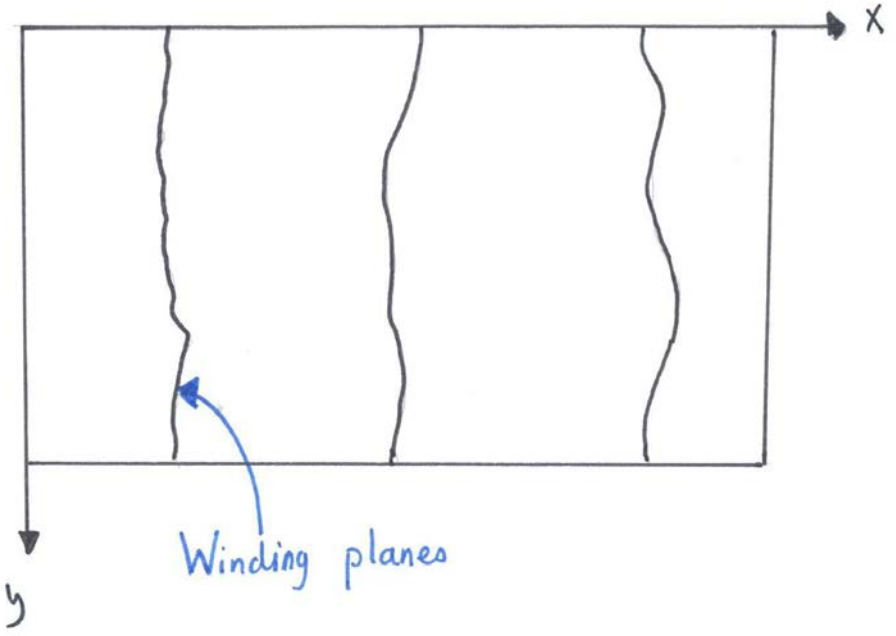
$$(\star K)_\mu = \partial_\mu\phi$$

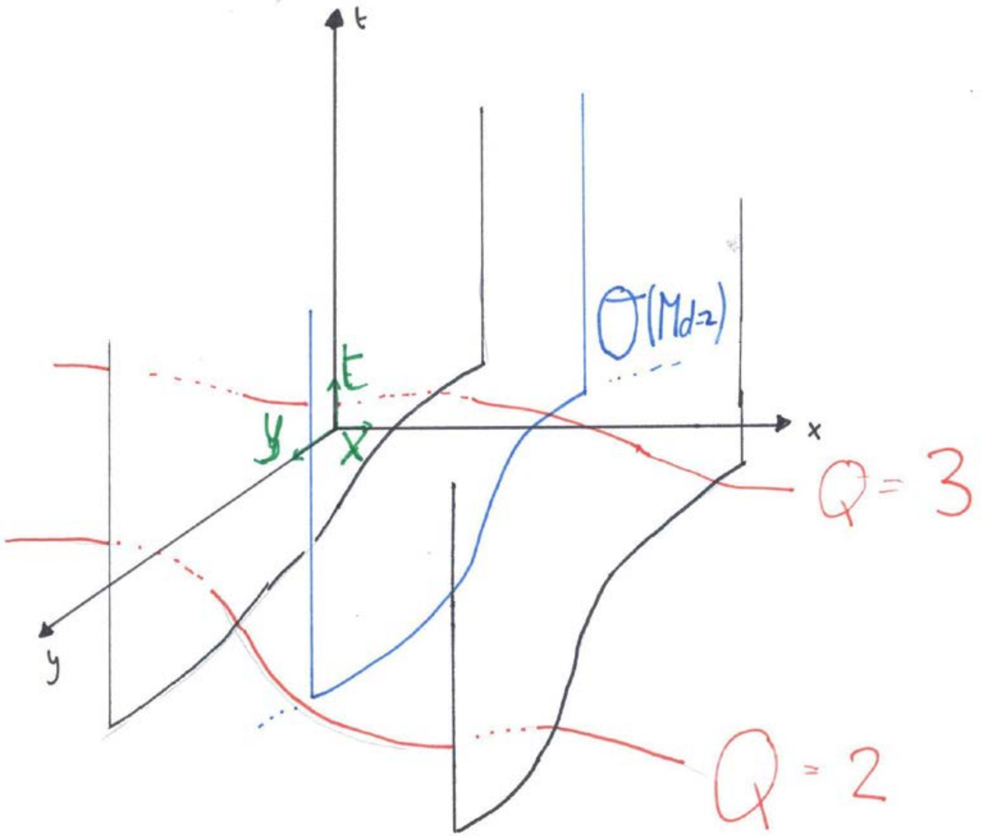
Both currents are conserved $d\star J = d\star K = 0$

Physical interpretation.

The objects charged under the higher-form symmetry are winding planes of the phase. There are (d-2) dimensional.

The charge operator is $Q = \int \star K = \frac{1}{2\pi} \int d\phi = n \in \mathbb{Z}$





Anomaly

The regular U(1) 0-form symmetry can be coupled to a background field as usual by promoting

$$\partial_\mu \phi \to D_\mu \phi \equiv \partial_\mu \phi - A_\mu$$

The higher-form current becomes

$$\star K = d\phi, \qquad \Rightarrow \qquad \star K = D\phi = d\phi - A$$

It is no longer conserved, but instead we have

$$d \star K = -F, \qquad\qquad F = dA$$

$\Rightarrow$ The symmetry is anomalous!

Framework

Consider a relativistic superfluid. It can be described by the low energy effective field theory [D.T. Son, '02]

$$S = \int d^dx P\left(\sqrt{-D_\mu\phi D^\mu\phi}\right) + \ldots$$

With

$$D_\mu\phi = \partial_\mu\phi - A_\mu$$

The U(1) current is given by

$$J_\mu = P' \frac{D_\mu\phi}{\sqrt{-D_\nu\phi D^\nu\phi}} + \ldots$$

Framework continued

Notice that the EFT

$$S = \int d^d x P\left(\sqrt{-D_\mu \phi D^\mu \phi}\right) + \ldots$$

Also enjoys a (d-2)-form symmetry $U(1)^{d-2}$ carried by the current $K_{\mu_1 \mu_2 \ldots \mu_{d-1}}$ given by

$$(\star K)_\mu = D_\mu \phi$$

In the presence of non-trivial background gauge fields $F = dA \neq 0$, the conservation of the higher form current is broken by an anomaly as

$$d \star K = -aF$$

Intermediate summary

At this stage we found that

1. We can distinguish two phases of a system without using the usual Landau paradigm that uses spontaneous symmetry breaking, but looking at the global symmetry that are different on both side (Ex: BKT phase transition)
2. When there is a U(1) symmetry that gets spontaneously broken, there is an emergent higher-form symmetry $U(1)^{(d-2)}$, which is anomalous.
3. This emergent higher-form symmetry has a mixed anomaly with the U(1) when coupled to a background gauge field.

Next goal

We want to show that there exists an (almost) converse statement : A system with $U(1) \times U(1)^{(d-2)}$ symmetry with a mixed anomaly

$$d \star K = -aF$$

contains a massless Goldstone boson transforming non-linearly in its spectrum.

$\Rightarrow$ SSB phases of system with abelian symmetries $\rightleftharpoons$ anomalies.

Goldstone theorem

An alternative to Goldstone's theorem

We start from the input that the theory has a global symmetry $U(1) \times U(1)^{(d-2)}$ with mixed anomaly.

$$\partial_\mu J^\mu = 0, \qquad \partial_{[\mu}(\star K)_{\nu]} = -aF_{\mu\nu}$$

The Fourier transform of the mixed correlator is constrained to be

$$\Pi_{\mu\nu} = \int d^d x e^{ipx} \langle \mathcal{T}((\star K)_\mu J_\nu) \rangle = f(p^2) p_\mu p_\nu + g(p^2) p^2 g_{\mu\nu}$$

The functions $f(p^2)$ and $g(p^2)$ are fixed by Ward identities to give

$$\Pi_{\mu\nu} = a \frac{p_\mu p_\nu - p^2 g_{\mu\nu}}{p^2}$$

Robust consequence of the anomaly

An alternative to Goldstone's theorem

The Källén-Lehmann representation of the time ordered correlator is

$$\Pi_{\mu\nu} = \int_0^\infty d\mu^2 \rho_{KJ}(\mu^2) \frac{p_\mu p_\nu - p^2 g_{\mu\nu}}{p^2 - \mu^2 + i\epsilon}$$

Comparing with

$$\Pi_{\mu\nu} = a \frac{p_\mu p_\nu - p^2 g_{\mu\nu}}{p^2}$$

We immediately conclude that there exists a massless state p^2=0 that is created by both current

$$\rho_{KJ}(\mu^2) = a\delta(\mu^2) + \dots$$

This is the same philosophy : Two ways to get massless phases : Spontaneous symmetry breaking or anomalies. But actually, only anomalies are needed.

Hydrodynamics

Relativistic hydrodynamics

The conserved currents (with a global U(1)) are

$$\partial_\mu T^{\mu\nu} = 0, \qquad\qquad \partial_\mu J^\mu = 0$$

Assume local thermal equilibrium : We can write the constitutive relations

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$
$$J^\mu = \rho u^\mu$$

Assumption: ε, P, ρ are functions of the hydrodynamic variables : T, μ.
We also need an equation of state $\mathrm{P} = P(\mu, T)$.

We can show (at ideal order) that

$$\partial_\mu (s u^\mu) = 0$$

Relativistic superfluids [Son, '00, Bhattacharya, Bhattacharyya, Minwalla, '11]

The superfluid variables are u^μ, μ, T, and ξ^μ with $P = P(T, \mu, \xi)$ and

$$\xi^\mu = -\partial^\mu \phi(x)$$

Goldstone mode (microscopic)

The equation of superfluid dynamics are

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^\mu = 0$$

In addition, the superfluid velocity satisfies a Josephson condition

$$u^\mu \xi_\mu = \mu$$

We add, as equation of superfluid dynamics

$$\partial_\mu \xi_\nu - \partial_\nu \xi_\mu = 0$$

ξ^μ is the gradient of a scalar

Our reformulation

We want to construct the complete hydrodynamics of a system with an (anomalous) $U(1) \times U(1)^{(d-2)}$ symmetry.

We should have a conserved energy-momentum tensor $T^{\mu\nu}$, a conserved current J^{μ} and an anomalous current $K^{\mu_1 \cdots \mu_{d-1}}$.

We take as hydrodynamic variables T, two chemical potentials $\mu, \tilde{\mu}$ and two vectors u^{μ} and h^{μ} such that

$$u^{\mu} u_{\mu} = -1, \qquad h^{\mu} h_{\mu} = 1, \qquad u^{\mu} h_{\mu} = 0$$

- u^{μ} is a timelike velocity vector that specifies the rest frame.
- h^{μ} specificies the the orientation of the planes charged under the higher-form symmetry.

Constitutive relations at zeroth order

The most general expressions are

$$T^{\mu\nu} = (\epsilon + p - \tau)u^\mu u^\nu + (p - \tau)\eta^{\mu\nu} + \tau h^\mu h^\nu + \gamma u^{(\mu} h^{\nu)}$$

$$J^\mu = \rho u^\mu + \sigma h^\mu$$

$$(\star K)^\mu = \tilde{\sigma} u^\mu + \tilde{\rho} u^\mu$$

← Josephson condition

with all scalar functions depending on $\mathrm{T}, \mu, \tilde{\mu}$.

The conservation equations are

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho$$

U(1) → $$\partial_\mu J^\mu = 0$$

$U(1)^{(d-2)}$ → $$\partial_{[\mu} (\star K)_{\nu]} = -a F_{\mu\nu}$$

Entropy current conservation and anomaly [Son, surowka, '09]

Consider the following combination of equations of motion

$$\Omega = u_\nu \partial_\mu T^{\mu\nu} + \mu \partial_\mu J^\mu + \tilde{\mu} u^\mu h^\nu \left(\partial_\mu \left(\star K\right)_\nu - \partial_\nu \left(\star K\right)_\mu \right)$$

Using the conservation equation, we got

$$\Omega = u^\mu h^\nu F_{\mu\nu} \left(\sigma - a\tilde{\mu}\right)$$

Using the constitutive equations and thermodynamic relation, it is easy to show that for the entropy current to be conserved in arbitrary flow, the scalar functions are uniquely fixed to be

$$\tau = \tilde{\mu}\tilde{\rho}, \qquad \gamma = a\mu\tilde{\mu}, \qquad \sigma = a\tilde{\mu}, \qquad \tilde{\sigma} = a\mu$$

Conclusion

Conclusion

- We have showed that

Spontaneous symmetry breaking of a continuous $U(1)$ symmetry → Emergence of an anomalous higher-form $U(1)^{(d-2)}$ symmetry

- We have showed that

Higher-form symmetry $U(1)^{(d-2)}$ together with a mixed anomaly → There is a gapless mode in the spectrum

Outlook

- Are all gapless phase protected by anomalies ?
- Non-Abelian case : Although higher-form symmetries are always Abelian, the 0-form symmetry that is spontaneously broken can be Non-abelian.
- Which phases are truly non-Landau, after generalized symmetries and their anomalies are taken into account ?

Thank you for your attention!

Back-up

Fractional Quantum Hall effect

This also works to distinguish certain FQH states. The low energy topological EFT for $\nu = 1/k$ Laughlin state is

$$S_{CS} = \frac{k}{4\pi} \int ada$$

Bianchi identity implies a magnetic $U(1)_m$ symmetric as

$$d(da) = 0 \rightarrow j^m_\mu = \epsilon_{\mu\nu\lambda} d^\nu a^\lambda$$

Electric symmetry : EoM is $da = 0$, so the following quantity looks conserved

$$j^e_{\mu\nu} = \epsilon_{\mu\nu\lambda} a^\lambda$$

Acts as $a_\mu \rightarrow a_\mu + \lambda_\mu$, but not any λ_μ is ok.

$$\delta S_{CS} = \frac{k}{2\pi} \underbrace{\oint f}_{=2\pi n} \cdot \int \lambda = n \cdot k \int k \Rightarrow \int \lambda = \frac{2\pi m}{k}, \qquad m = 0, 1, \ldots, k-1$$

Electric symmetry is $Z_k^{(1)}$, so different fillings have different symmetries.

2d SSB

Mermin-Wagner theorem: Continuous symmetry cannot be spontaneously broken at finite temperature in two dimensions. Long-range fluctuations can be created with little energy cost and since they increase the entropy they are favored.

But, the 2d XY model does exhibit a phase transition, whose phase is not distinguished by an order parameter but by correlation functions.

Low T : Pairs of topological defects. Energy cost scales like the separation between theirs cores, and at low T, no defect will have sufficient energy to move far away from its antidefect partner.

Hight T: Pairs become more prolific, and defects within a pair become further separated from their partners. At some T, the average separation becomes as large as separation between pairs. Single excitation freely roam the system.

2d SSB suite

The energy of a single isolated vortex is proportional to

$$E_{\text{one-defect}} \propto ln(L/\xi)$$

Where L is the linear system size and ξ its coherence length. There are about L^2 /ξ^2 ways to put an object of area ξ^2 into a system with area L^2. So the entropy is

$$S_{\text{one-defect}} \approx k_B ln(L^2/\xi^2)) \approx 2k_B ln(L/\xi)$$

It means that $$F_{\text{one-defect}} = E - TS \approx (J - k_B T) ln(L/\xi)$$

Where J is an energy scale that is model dependant. At low T, the energy cost of creating a vortex is bigger that the entropy gain, and isolated vortex will not occur. At sufficiently high T, isolated defects proliferate.