A hydrodynamical description for transport in the strange metal phase of cuprates

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Based on work with
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Cuprates

- Layers of CuO$_2$ planes bounded by rare earths
- Superconductivity and the most part of exotic properties happen on the CuO$_2$ plane \(\rightarrow\) 2D materials
- Universal properties despite many different compounds
- Among High-\(T_c\) superconductors Bi-2201 has a relatively low critical temperature even at optimal doping \(\Rightarrow\) ideal to test low \(T\) properties of the normal phase
Cuprates have almost the same Temperature vs doping (concentration of rare earth) phase diagram, characterized by many intertwined phases appearing at the same time.
• QCP is supposed to affect the properties of the strange metal phase:
  ▶ transport coefficients should assume simple scaling laws
  ▶ *Strong coupling*: no well defined quasi-particles.
The Resistivity and Hall angle issue

- In normal Fermi liquid (magnetic field perpendicular to $\text{CuO}_2$ planes)
  \[\rho_{xx} \sim T^2, \quad \cot \theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2\]

- In most of the cuprates
  \[\rho_{xx} \sim T, \quad \cot \theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2\]

- Actually in Bi-2201 is known that $\cot \theta_H \sim T^{1.5}$
Other transport coefficients are less known

• Some of them are just dominated by lattice vibration
  ▶ $\kappa_{xx}$ has an 80 % of lattice phonon contribution

• Transverse transport coefficients are independent of phonons contribution (typically very small signal)
  ▶ The Nernst coefficient $N$ ([Wang, 2006] for a review)
  ▶ The thermal Hall conductivity $\kappa_{xy}$ (measured in LSCO [Grissonnanche, 2019] and in YBCO [Zhang, 2000][Matusiak, 2009])
  ▶ Magnetoresistance typically $B^2$ suppressed
More orderings discovered recently

- Charge-density wave (CDW) order appears to be a ubiquitous feature of cuprate superconductors.
- Our material, Bi$_2$Sr$_2$CuO$_6$:  
  - 2D CDW confirmed (by X-ray diffraction) to extend to optimal and over-doped region [Peng 2018],
  - low critical temperature ($T_c \sim 10 - 33$ K).
What are charge density waves?

- Peierls (1955) suggested periodic distortion of 1D lattice can lower total energy.
- Start with first Brillouin zone $k = \pm \pi/a$ half filled.
- CDW distortion $\rightarrow$ new superlattice of spacing $2a$. New first Brillouin zone band gap at $k = \pm \pi/2a$.
- Gain in creating energy gaps can overcome loss of lattice distortion.

Incommensurate CDW $\rightarrow$ broken translation invariance.
CDW and pinning

As soon as the translation SB is pseudo-spontaneous (Goldstone Bosons have a small mass) the AC conductivity can have an off-axes peak [Fukuyama-Lee-Rice ’78, Delacretaz 2017]

\[ \sigma(\omega) = \sigma_0 + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2} \]

- for \( \omega_0^2 > \Omega^3/(\Gamma + 2\Omega) \) there is an off-axes peak
- can the Drude to off axes peak originate from the same mechanism?

Figure: Experimental BiSCO conductivity from [Tsvetkov 1997]
CDW not only affects the conductivity

- Usually the enhancement in the Nernst effect at low $T$ was attributed to fluctuating superconductivity
- [Cyr-Choinière 2009] found a relation between $T_{CDW}$ and the enhancement temperature

$T_{\nu}$ is the temperature at which one recovers a Fermi Liquid expectation ($T_{\nu} \sim 2T_{CDW}$)

- CDW affects the Nernst signal also at fluctuating level
Where do we stand?

- Can one mechanism take into account consistently all the thermo-electric transport coefficients?
- Many intertwined phases ⇒ difficult to uncover
- We need a metallic behavior
- Strange metals are strongly coupled by nature

Hydrodynamics might come to help
Hydrodynamics as an EFT

• At large length and time scales, only a small number of DOFs survive to become hydrodynamic modes
  ▶ If no spontaneously broken symmetries: (almost)-conserved currents.

• EOMS are determined by symmetries. Eg in a the relativistic charged fluid there are two conserved currents:

\[ \partial_\mu J^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0 \]

• Local thermal equilibrium: everything is function of \( \mu(x), T(x) \) and \( u^\mu(x) \) \( \Rightarrow \) gradients expansion:

\[ J^\mu = n u^\mu + \mathcal{O}(\partial), \quad T^{\mu\nu} = (n + p) u^\mu u^\nu - p g^{\mu\nu} + \mathcal{O}(\partial) \]

Eventually one solves the EOMs order by order to find the relevant observables
Hydrodynamics VS Fermi Liquid

- Fermi liquid has well-defined quasi-particles around the Fermi Surface, which interact weakly.
- To see hydrodynamics effect the interaction time must be the smallest scale in the system.

Hydrodynamics is the correct EFT to describe strange metals: strongly coupled materials where the relevant long-lived DOF are the (almost)-conserved currents.
A unified hydrodynamic picture?

Let us play simple and start with DC transport coefficients.
Experiment (Please be kind here!)

- We want to measure the temperature $T$ and magnetic field $B$ dependence of all the thermo-electric transport coefficients
- We will restrict to transverse or electric transport coefficients to avoid phonons contribution (no $\kappa_{xx}$)
  - The electric conductivity $\rho_{xx}$
  - The Hall angle $\cot \theta_H = \frac{\rho_{xy}}{\rho_{xx}}$
  - The magnetoresistence $\frac{\rho_{xx}(B) - \rho_{xx}(0)}{\rho_{xx}(0)}$
  - The thermal Hall conductivity $\kappa_{xy}$
  - The Nernst signal $N$
- Many coexisting phases $\Rightarrow$ we need to properly define the temperature range where the picture is supposed to be valid
$B$ dependence of the DC transport coefficients

- For $T < 20$ K the Nernst starts to deviate from linearity $\Rightarrow$ Vortex effect [Wang 2006]
- For $T > 20$ K the $B$ dependence is the one expected for a parity invariant system
$T$ dependence of the DC transport coefficients upper bound

- **Estimation of** $T_\nu$: the point where $N/T$ deviates from linearity at high temperature: $T_{CDW} \sim T_\nu/2 = 65$ K [Cyr-Choinière 2009]
- In accordance with [Peng, 2018]
$T$ dependence of the DC transport coefficients

- Relevant temperature interval $20 \, \text{K} < T < 65 \, \text{K}$
Summary of experimental results

- **How do experimental parameters depend on $T$ and $B$?**
  - $\rho_{xx} \sim B^0 T$ as expected for strange metals.
  - $\Delta \rho/\rho \sim B^2 T^{-4}$
  - $\cot \theta_H \sim B^{-1} T^{1.5}$ as expected in Bi-2201 but different from other materials (YBCO $\cot \theta_H \sim B^{-1} T^2$).
  - $\kappa_{xy} \sim BT^{-3}$.
  - $N \sim BT^{-2.5}$
Hydrodynamics with broken continuous symmetries and dissipation

The breaking of translations can be pseudo-spontaneous

- Momentum dissipation rate $\Gamma$: coupling to external lattice
- Phase relaxation $\Omega_1$ of the GBs: present as soon as translations are explicitly broken [Amoretti 2018]
- The magnetic fields $F_{xy} = B$ enters only as an external field via the Lorentz term

The total EOMs:

$$\partial_t (n, s) + \partial_i (J^i, Q^i / T) = 0 ,$$

$$\partial_t \pi^i + \partial_j T^{ji} = F^{ij} J_j - \Gamma \pi^i - k_0^2 G \phi^i ,$$

$$\partial_t \phi_a + \partial_i J^{i\phi_a} = -\Omega_1 \phi_a .$$
Constitutive relations

The only missing step is to provide constitutive relations for the currents $J_i$, $Q_i/T$, $T^{ij}$ and $J_{\phi_a}^i$ to first order in the gradients expansion around the equilibrium configuration $T + \delta T, \mu + \delta \mu$:

\[
\begin{align*}
\frac{Q^i}{T} &= sv^i - \alpha_0 \left( \partial^i \delta \mu - F^{ij} v_j \right) - \frac{\bar{K}_0}{T} \partial^i \delta T - \gamma_2 \partial^i \theta_1 , \\
J^i &= nv^i - \sigma_0 \left( \partial^i \delta \mu - F^{ij} v_j \right) - \alpha_0 \partial^i \delta T - \gamma_1 \partial^i \theta_1 , \\
T^{ij} &= (n \delta \mu + s \delta T - (G + K) \chi_1 \theta_1) \delta^{ij} - G \chi_2 \theta_2 \epsilon^{ij} \\
&\quad - \eta \left( \partial^i v^j + \partial^j v^i - \partial_k v^k \delta^{ij} \right) - \zeta \partial_k v^k \delta^{ij} + \gamma_1 B \theta_2 \delta^{ij} \\
J_{1}^{i} &= -v^i - \gamma_1 \left( \partial^i \delta \mu - F^{ij} v_j \right) - \gamma_2 \partial^i \delta T - \xi_1 \chi_1 \partial^i \theta_1 + \xi_2 \chi_2 \epsilon^{ij} \partial_j \theta_2 , \\
J_{2}^{i} &= \epsilon^{ij} J_{1}^{j} ,
\end{align*}
\]

- Transport coefficients
- Susceptibilities
Constraints

• Typical constraints for charged fluid:

\[ \sigma_0, \bar{\kappa}_0, \eta, \Gamma, \Omega_1 \geq 0, \quad \bar{\kappa}_0 \sigma_0 - T \alpha_0^2 \geq 0. \]

• Special to CDW: \( \xi_1 > 0. \)

• This subsequently leads to bounds on \( \gamma_1 \) and \( \gamma_2 \):

\[ \left( \gamma_1^2, \gamma_2^2 \right) \leq \left( \sigma_0, \frac{\bar{\kappa}_0}{T} \right) \min \left[ \frac{\xi_1}{K + G}, \frac{\Omega_1}{\chi_{\pi\pi} \omega_0^2} \right]. \]

• We will assume \( \gamma_{1,2} \) are small enough to be treated as vanishing.

• If we assume a relativistic covariant fixed point then

\[ \alpha_0 = -\frac{\mu \sigma_0}{T}, \quad \bar{\kappa}_0 = \frac{\mu^2 \sigma_0}{T}. \]
The Martin-Kadanoff method

Having the modified EOMs and the constitutive relations one can apply the Martin-Kadanoff procedure

- One can cast the EOMs in the following way ($q_A$ are the relevant fields, $s_A^0$ are the sources):

$$
\partial_t q_A(t, \vec{k}) + M_A^C(\vec{k}, B)s_C(t, \vec{k}) = \chi_B^B s_B^0(\vec{k}).
$$

- The retarded Green’s function can eventually be computed

$$
- \left( I_6 + i\omega \left( -i\omega I_6 + M\chi^{-1} \right)^{-1} \right) \chi.
$$
Conductivities at low $B$

- Taking the DC transport coefficients to lowest order in $B$:
  - Charge resistivity: $\rho_{xx} = \frac{1}{\sigma_0 + \tilde{\sigma}} + \mathcal{O}(B^2)$.
  - Magnetoresistance: $\Delta \rho / \rho = B^2 \frac{\sigma_0^3}{n^2} \frac{\tilde{\sigma}}{(\sigma_0 + \tilde{\sigma})^2} + \mathcal{O}(B^4)$.
  - Thermal Hall conductivity:
    $$\kappa_{xy} = -B T \frac{\tilde{\sigma}^2 s}{n^4} \left( ns - 2 \frac{\mu \sigma_0 n^2}{T \tilde{\sigma}} \right) + \mathcal{O}(B^3).$$
  - Hall angle: $\cot \Theta_H = \frac{n}{B \tilde{\sigma}} \frac{1 + \frac{\sigma_0}{\tilde{\sigma}}}{1 + 2 \frac{\sigma_0}{\tilde{\sigma}}} + \mathcal{O}(B)$.
  - Nernst coefficient: $N = \frac{B}{n^2 (\sigma_0 + \tilde{\sigma})^2} \sigma_0 (s + \frac{\mu}{T}) + \mathcal{O}(B^3)$.

- DC conductivities are a sum of incoherent and relaxation conductivities
  $$\sigma_{DC} = \sigma_0 + \tilde{\sigma} \quad \text{with} \quad \tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_1}{\Omega_1 \Gamma + \omega_0^2}.$$

- Only four variables $\sigma_0$, $\tilde{\sigma}$, $n$ and $s$. But we measure five observables - system overconstrained.
Determining the hydrodynamic variables

- **What does experiment imply for our hydrodynamic variables?**
  - Consistency requires \( \rho_{xx} \) dominated by \( \sigma_0 \) at low \( T \) i.e.
    \[
    \rho_{xx} \sim \frac{1}{\sigma_0} \sim T,
    \]
  - and
    \[
    \cot \Theta_H \sim \frac{n}{B\tilde{\sigma}} \sim T^{1.5}.
    \]
  - Using \( \Delta\rho/\rho \sim T^{-4} \) fixes
    \[
    n \sim T^{1.5} \quad \text{and} \quad \tilde{\sigma} \sim T^0.
    \]
  - Finally \( s \) is given through \( \kappa_{xy} \)
    \[
    \kappa_{xy} \sim \mu B \frac{\sigma_0 \tilde{\sigma}}{n^2} s \sim T^{-3} \quad \Rightarrow \quad s \sim T.
    \]
  - \( s \) is in accordance with specific heat measurement on our sample and on YBCO [Loram 1991]
Recovering the Nernst behavior

- The Nernst coefficient behaves as
  \[ N \sim \frac{\mu B \tilde{\sigma}}{nT} \sim \frac{\mu}{T \cot \Theta_H} \sim T^{-2.5}. \]

- The temperature range where the scaling agrees is exactly the one predicted from other principles (vortices at low \( T \) and \( T_{\nu}/2 \) at high \( T \))
Outlook

- This is a consistency check of the validity of hydro
  - We can not say anything on what is dominating $\tilde{\sigma} \Rightarrow$ need for precision spectral measurements
  - If hydro is valid down to low $T$ the Drude to off-axes peak should be explained within the same picture

- Other cuprates have different temperature scalings for the transport coefficients (eg Hall angle and $\kappa_{xy}$ in YBCO)
  - CDW order is measured almost in every cuprates $\Rightarrow$ try to find a consistent picture
  - Is hydro a valid description in different point of the phase diagram?
Thank You